

Stuksgewyse Integrasie

Evaluate the following integrals using integration by parts:

$$\int x e^{3x} dx \qquad \frac{x e^{3x}}{9} + \frac{e^{3x}}{9} + k$$

$$\int x \sin(5x) dx \qquad -\frac{x}{5} \cos 5x + \frac{1}{25} \sin 5x + k$$

$$\int x^2 \ln(x) dx \qquad \frac{x^3 \ln x}{3} - \frac{x^3}{9} + k$$

$$\int x^2 e^{-5x} dx \qquad \frac{x^2 e^{-5x}}{5} - \frac{2x e^{-5x}}{25} - \frac{2e^{-5x}}{125} + k$$

$$\int x^2 \cos(3x) dx \qquad \frac{x^2}{3} \sin 3x + \frac{2x}{9} \cos 3x - \frac{2}{27} \sin 3x + k$$

$$\int (x^2 + 2x + 1) e^{7x} dx \qquad \text{Solution follows after table}$$

$$\int \arctan(x) dx \qquad x \arctan x - \frac{1}{2} \ln|1 + x^2| + k$$

Evaluate

$$\int (x^2 + 2x + 1)e^{7x} dx$$

Let

$$u = x^2 + 2x + 1$$

$$dv = e^{7x} dx$$

Then

$$du = (2x + 2) dx$$

$$v = \frac{1}{7}e^{7x}$$

Hence,

$$uv - \int v du = \frac{1}{7}(x^2 + 2x + 1)e^{7x} - \int \frac{1}{7}(2x + 2)e^{7x} dx$$

Let

$$U = 2x + 2$$

$$dV = \frac{1}{7}e^{7x} dx$$

Then

$$dU = 2 dx$$

$$V = \frac{1}{49}e^{7x}$$

Hence,

$$= \frac{1}{7}(x^2 + 2x + 1)e^{7x} - \left[\frac{(2x + 2)e^{7x}}{49} - \int \frac{2e^{7x}}{49} dx \right]$$

$$= \frac{1}{7}(x^2 + 2x + 1)e^{7x} - \frac{1}{49}(2x + 2)e^{7x} + \frac{2}{343}e^{7x} + C$$

$$\int (x^2 + 2x + 1)e^{7x} dx = \frac{e^{7x}(49x^2 + 84x + 37)}{343} + C$$