α-MATHEMATICS

Alpha Mathematics

FINAL EXAM PAPER

October 2024 **Time: 3 hours**

Grade 12 **Total: 200 marks**

INSTRUCTIONS AND INFORMATION

Carefully read through the following instructions before answering the examination paper:

1. Answer all 10 questions on this examination paper.
2. Write your name and ID number on the front page or the examination paper.
3. Non-programmable calculators may be used, unless otherwise indicated at a specific question.
4. Unless indicated otherwise, all answers, where applicable, must be given correct to **TWO decimal places.**
5. The diagrams in the examination paper are not necessarily drawn to scale.
6. All angles are given in radians. Answers must be given in radians where applicable.
7. This examination paper consists of a front page, **10 question** and a formula sheet.
8. Question 1 consists or **15** multiple choice questions. Answer it on the answer sheet. This answer sheet is at the front or the paper. **Do not remove the answer sheet**.
9. For all questions, all necessary calculations must be shown clearly. The correct answer on its own will not necessarily lead to full marks.
10. Additional writing space is provided at the end of this examination paper. Clearly indicate if you made use of this to complete a question.
11. Write neatly and legibly.

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QUESTION 1 **[30 MARKS]**

- Answer this question **on the answer sheet,** which is attached to the front, by making a X (cross) on A, B, C or D. These questions count 2 marks each.
- Please **DO NOT** detach this page from the paper.

1.1 When a rational function is sketched, which of the following statements is NOT true:

(A) There are vertical asymptotes at the x-values that make the function undefined.

(B) There is an oblique asymptote when the degree of the nominator equals one more than that of the denominator.

(C) There can be a horizontal and an oblique asymptote.

(D) The function can intercept the horizontal asymptote.

1.2 What is calculated repeatedly with the Newton-Raphson method?

(A) x- intercepts of the tangents.

(B) y- intercepts of the tangents.

(C) The gradient of the tangents.

(D) The discriminant of the function.

1.3 The value of i^{15} equals

A) 1

B) -1

C) i

D) $-i$

1.4 Given: $\sqrt{5}-2$ is a zero of a function. Which of the following will then be a factor of the function?

(A) x^2+4x-1

(B) $x-2-\sqrt{5}$

(C) x^2-4x-1

(D) $x-2+\sqrt{5}$

1.5 Determine the coefficient of x^4 in $(x-2)^{10}$.

(A) $-13\,440$

(B) $13\,440$

(C) $-8\,064$

(D) $8\,064$

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1.6 For which value of m will the two equations: $mx - y = 3$ and $6x - 2y = 4$ have infinite solutions?

(A) $m = 3$

(B) $m = -3$

(C) $m = -12$

(D) for no value of m.

1.7 $\int sinx.cosx\,dx =$

(A) $-cosx.sinx + k$

(B) $cos^2x + sin^2x + k$

(C) $\frac{-1}{4}cos(2x) + k$

(D) $\frac{1}{2}sin(2x) + k$

1.8 If $g(x) = \cos x$ and $f(x) = \arccos x$, calculate $f\left(\mathrm{g}\left(\frac{4\pi}{3}\right)\right)$

(A) $\frac{4\pi}{3}$

(B) $\frac{2\pi}{3}$

(C) $\frac{\pi}{3}$

(D) $-\frac{4\pi}{3}$

1.9 If $a = i + j + k$ and $b = i + 2j + 3k$, determine a vector that is perpendicular to $a + b$ and $a - b$.

(A) $i - 2j + k$

(B) $2\mathrm{i} + 3\mathrm{j} + 4\mathrm{k}$

(C) $-2i - 4j - 2k$

(D) $-2i + 4j - 2k$

1.10 Study the function $f(x) = \begin{cases} \boldsymbol{x+1} & \text{if} \quad x < 0 \\ \boldsymbol{-1} & \text{if} \quad 0 \leq x < 2 \\ \boldsymbol{1} & \text{if} \quad x = 2 \\ \boldsymbol{x-3} & \text{if} \quad x > 2 \end{cases}$

Is the function continuous at $x = 2$? If not, give the type discontinuity.

(A) sprung

(B) removable

(C) asymptotic

(D) f is continuous

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1.11 Which notation is incorrect:

(A) If $f(x) = 20x^4 \; then \; f'(x) = 80x^3$

(B) $D_y\,[20x^4] = 80x^3$

(C) If $y = 20x^4, then \; \frac{dy}{dx} = 80x^3$

(D) $\frac{d}{dx}[20x^4] = 80x^3$

1.12 Determine , $\frac{dy}{dx}$ at $x = -1$ If $y = xy + x^2 + 1$

(A) $-\frac{1}{2}$

(B) $\frac{1}{2}$

(C) -1

(D) 2

1.13 Which one of the following equals $\int \frac{1}{\sqrt{25-x^2}}$

A) $\arcsin\frac{x}{5} + k$

B) $\arcsin x + k$

C) $\frac{1}{5}\arcsin\frac{x}{5} + k$

D) $\frac{1}{5}\arcsin x + k$

1.14 $\int \left(\sin 2x.\frac{1}{2\sqrt{x}} + 2\sqrt{x}\cos 2x\right)dx =$

A) $\cos 2x.\frac{2\sqrt{x^3}}{3} + k$

B) $-\frac{\sin 2x.x^{\frac{3}{2}}}{3} + k$

C) $\sqrt{x}\sin 2x + k$

D) Not determinable

1.15 The value of the derivative of $y = \frac{\sqrt[3]{x^2+8}}{\sqrt[4]{2x+1}}$ at $x = 0$ is

A) -1

B) $-\frac{1}{2}$

C) 0

D) $\frac{1}{2}$

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Answer the following questions **on the exam paper** on the lines provided below each question. Clearly indicate if you use the additional writing space at the end of the paper to complete a question.

QUESTION 2 **[23 MARKS]**

2.1 Given: $\boldsymbol{f(t) = \frac{A}{t} + \ln\left(t^{-\frac{1}{2}}\right)}$

(a) Determine the value of A, rounded to 5 decimal digits, if $f(2) = 0{,}1534$. (3)

(b) Determine $f'(t)$ in terms of t if A = 1. (4)

2.2 Consider the equation $\ln(x-1) = 1 + \ln(3x+2)$.

(a) Simplify the equation by using the laws of logarithms and rewrite it in the form $x = \cdots$ (4)

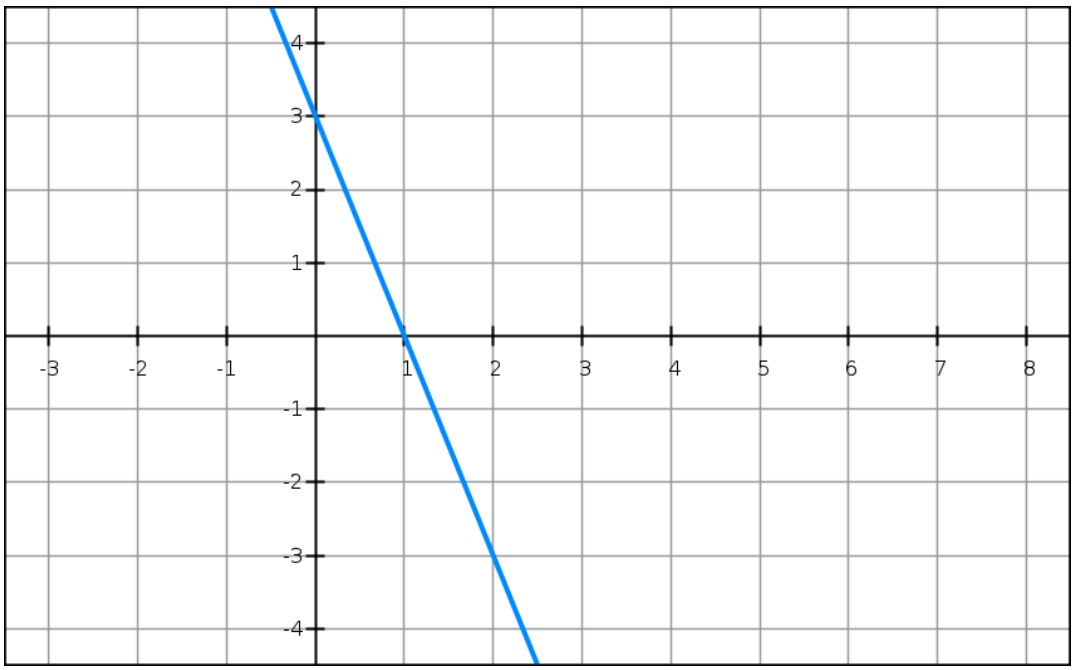
(b) Hence show clearly why this equation has no real values. (3)

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2.3 Consider the following graphs: $y = |x - 4| - 1$ and $3x + y = 3$.

(a) Calculate the x-value(s) of the intercept(s) of the graphs. (5)

(b) The sketch below shows the graph of $3x + y = 3$. Sketch on the same set of axes the graph of $y = |x - 4| - 1$. Show the intercept with the axes, the salient point and the point(s) calculated in Question 2.3(a) on your graph. (4)



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QUESTION 3 **[22 MARKS]**

3.1 Given $P(x) = 2x^4 + 4x^3 - 9x^2 - 16x + 4$ that has $x + \frac{2-\sqrt{6}}{2}$ as a factor.
Factorise $P(x)$ completely in $\mathbb{R}[x]$. (7)

3.2 (a) Write z_1 and z_2 in polar form if $z_1 = 4(-1 + i)$ and $z_2 = 2(-\sqrt{3} - i)$.
(Give all answer in surd form and in terms of π where necessary.) (4)

(b) Use de Moivre's theorem and simplify $\frac{(z_1)^2}{(z_2)^3}$.
Give the answer(s) in rectangle form. (5)

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3.3 Calculate all possible values of z if $z^4 = 16$.
Give the answer(s) in polar form. (6)

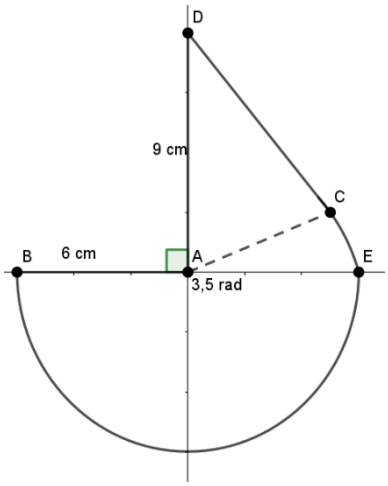
QUESTION 4 **[19 MARKS]**

4.1 Expand $(1 + 3x)^{-\frac{1}{3}}$, up to and including x^3, and simplify the coefficients. (6)

4.2 Use the binomial theorem and write down the first three terms of $\left(\frac{x}{2} - \frac{2}{x^2}\right)^7$.
Simplify the coefficients. (6)

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4.3 The sketch shows circular arc BEC with a radius of 6 cm and an angle of 3,5 radians. $B\hat{A}D = \frac{\pi}{2}$. Determine the circumference of BECDAB. (7)



QUESTION 5 **[13 MARKS]**

5.1 (a) If $\boldsymbol{a} = k\boldsymbol{b}$ with vectors $\boldsymbol{a} = (9; -6; 3)$ and $\boldsymbol{b} = (6; -4; 2)$, determine the value of k. (2)

(b) Are $\boldsymbol{a}$ and $\boldsymbol{b}$ parallel to each other or perpendicular to each other? (1)

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(c) Determine the unit vector of $\boldsymbol{b} = (6; -4; 2)$ in its simplest form. (4)

5.2 Consider the vectors $u_1 - 2u_2 + 3u_3$ and $3v_1 - 2v_2 + v_3$. Determine the angle between the two vectors. (6)

QUESTION 6 **[19 MARKS]**

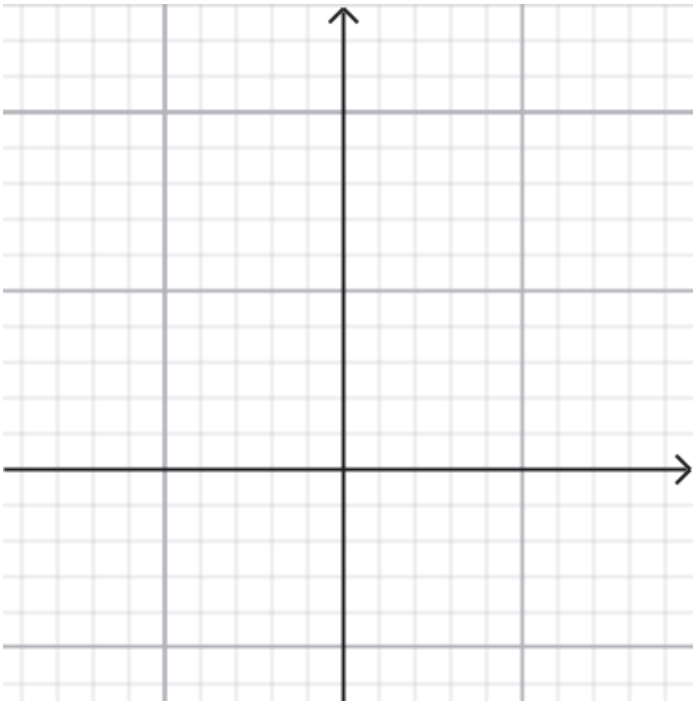
6.1 Consider the function $f(x) = \cos 2x$.

(a) How will you restricy the domain of so that the inverse will also be a function? (1)

(b) Write down the coordinates of the starting and endpoints of f whwn it is restricted as in question 6.1 (a) (2)

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(c) Use the coordinates that you have written down in 6.1 (a) and (b), and sketch the inverse function. Clearly show the coordinates of the intercepts with the axes as well as the starting and the end points. (4)



6.2 Give all possible solutions of θ if:

(a) $\theta = 2\arctan(-1)$ (1)

(b) $\sin\frac{\theta}{2} = -1$ (3)

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6.3 Use mathematical induction and prove that

$13 + 6 - 1 + \cdots + (20 - 7n) = \frac{(33n - 7n^2)}{2}$ for all $n \in \mathbb{N}$. (8)

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QUESTION 78 **[21 MARKS]**

7.1 Determine the derivative of the following: (Answers needn't be simplified)

(a) $\frac{d}{dx}(\log(\cot 4x^2))$ (4)

(b) $g'(x)$ if $g(x) = \cos\left(\frac{x}{5}\right) \times \arccos\sqrt{5x}$ ((5)

8.2 Use implicit differentiation and determine $\frac{dy}{dx}$ if $x^2y + \ln y = x^2 - 4$ (5)

8.3 Consider the graph with equation $y = \frac{1+e^{2x}}{1+3x}$ which has one stationary point.

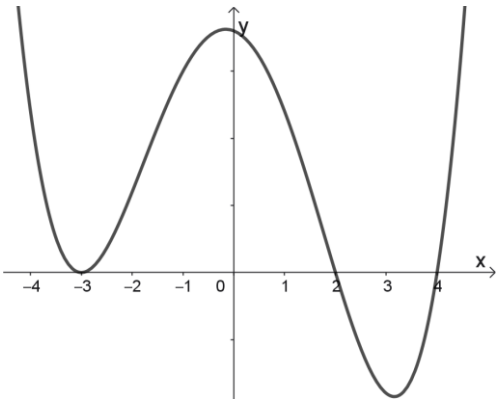
(a) Show that the x-value of this stationary point can be determined by solving the equation $x = a + be^{-2x}$.
Determine the value of $\boldsymbol{a} + \boldsymbol{b}$. (You don't have to solve for x.) (6)

(b) Give only the method that will be used to find x. (1)

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QUESTION 8 **[17 MARKS]**

8.1 The sketch beneath shows the graph of $y = f'(x) = x^4 - 19x^2 - 6x + 72$.



(a) For which values of x will f have stationary points? (3)

(b) Hence determine for each stationary point of **Question 8.1(a)** whether this point is a point of inflection, a minimum or maximum turning point.
Fully motivate your answer. (5)

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8.2 Use a Riemann sum and determine the value of $\boldsymbol{m}$ if $\int_{-1}^{2}(x^2 + \boldsymbol{m})\,dx = 21$. (9)

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QUESTION 9 **[21 MARKS]**

Determine the following integrals:

9.1 $\int \left(2x^2 + 2 + \frac{2}{x}\right) dx$ (3)

9.2 $\int (\tan^2 x + \cos^2 x) dx.$ (5)

9.3 $\int \frac{3x^2+3x+7}{(x^2+1)(x-2)} dx$ by using partial fractions. (8)

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9.4 $\int (\text{cosec}^3\, 2\theta \cot 2\theta)\, d\theta$ (5)

QUESTION 10 **[15 MARKS]**

Given that $f(x) = \arctan 2x$.

10.1 (a) Determine $f'(x)$ (2)

(b) Use Newton-Raphson's method and determine the x-coordinate of the intercept of f and $g(x) = x - 2$, correct to 4 decimal digits.
(Use $x = 2$ as first approximation.) (5)

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10.2 Use piece-wise integration, **Question 10.1 (a)** as well as the fact that $\frac{4x^2}{4x^2+1} = 1 - \frac{1}{4x^2+1}$ and determine $\boldsymbol{a}$ **and** $\boldsymbol{b}$ if: $\int_0^{\frac{\sqrt{3}}{2}} x.arctanx\,dx = a\pi + b\sqrt{3}$ (8)

TOTAL: 200 marks

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EXTRA SPACE

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