α-WISKUNDE

Alpha Wiskunde Graad 11 / ***Alpha Mathematics Grade 11***

Halfjaar eksamen 2019 / ***Half year examination 2019***

MEMORANDUM

Totaal / ***Total*****: 165 punte /** ***marks***

Hierdie memorandum bestaan uit 9 bladsye. /
This memorandum consists of 9 pages.

Vraag / *Question* 1 **[20 punte / *marks*]**

NR. NO	ANTWOORD *ANSWER*	BEREKENINGE (nie vir nasien doeleindes nie) *CALCULATIONS (not for marking purpose)*	PUNTE *MARKS*
1.1	A	$y = \|x - 2\| - 1$	**2**
1.2	C	$2x - 1$ is 'n faktor, dus $x = \frac{1}{2}$ moet 'n nulpunt wees. $4\left(\frac{1}{2}\right)^3 - 4\left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right) + 1 = 0$	**2**
1.3	B	$\left(\frac{1}{2}\right)^{-5}(1 - 2x)^{-5}$ $\therefore \|-2x\| < 1$ $\|x\| < \frac{1}{2}$	**2**
1.4	C	Altyd 1 meer as die mag van die hakie. Dus 5 terme vir 'n binominale uitbreiding.	**2**
1.5	B	$x = 3x + 1$ $\quad -x = 3x + 1$ $x = -\frac{1}{2}$ (as $x \geq 0$) $\quad x = -\frac{1}{4}$ (as $x < 0$)	**2**
1.6	D	$-\frac{\pi}{2} \leq 4x \leq \frac{\pi}{2}$ $x \in \left[-\frac{\pi}{8}; \frac{\pi}{8}\right]$	**2**
1.7	D	$\binom{9}{5}\left(\frac{1}{5}a\right)^{9-5}(10b^2)^5 = 20160a^4b^{10}$ $\therefore K = 20160$	**2**
1.8	D	Geeneen	**2**
1.9	A	$f(0) = 0 - 4 = -4$ $f(5) = 5 - 4 = 1$ $f(10) = -10 + 6 = -4$ $-4 + 1 - 4 = -7$	**2**
1.10	B	Onthou irrasionale en komplekse wortels kom in pare. $ax^7 + bx^5 + cx^3 + dx + e = 0$ het 'n maksimum van 7 wortels. Moontlike kombinasies: 7 reële wortels 2 kompleks; 2 kompleks; 2 kompleks; 1 reëel 2 irrasionaal; 2 irrasionaal; 2 irrasionaal; 1 reëel 2 kompleks; 2 irrasionaal; 3 reëel ens... Dus is daar ten minste een reële wortel.	**2**

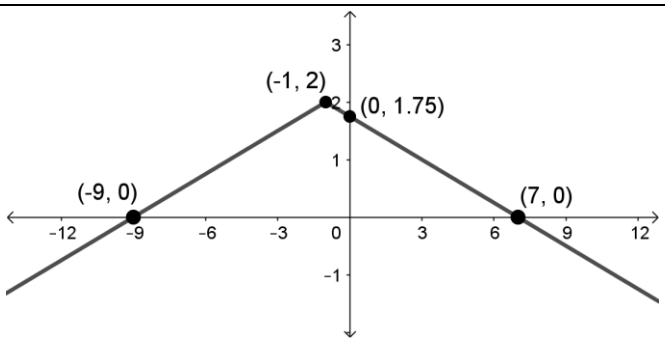
Vraag / *Question* 2 **[21 punte / *marks*]**

NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
2.1	$\frac{3x^2-2x+3}{(x^2+1)^2} \equiv \frac{Ax+B}{x^2+1} + \frac{Cx+D}{(x^2+1)^2}$ $3x^2-2x+3 \equiv (Ax+B)(x^2+1)+(Cx+D)$ $\equiv Ax^3+Bx^2+Ax+B+Cx+D$ $\equiv x^3(A)+x^2(B)+x(A+C)+(B+D)$ $A=0 \quad B=3 \quad A+C=-2 \quad B+D=3$ $C=-2 \quad D=0$ $\therefore \frac{3x^2-2x+3}{(x^2+1)^2} \equiv \frac{3}{x^2+1} + \frac{-2x}{(x^2+1)^2}$	✓ Ontbind in parsiële breuke ✓ Vermenigvuldig met KGV ✓ Vereenvoudig ✓ Groepeer volgens magte van x ✓ A ✓ B ✓ C ✓ D ✓ Finale antwoord **[9 punte / *marks*]**
2.2	$\frac{1}{1\times4}+\frac{1}{4\times7}+\frac{1}{7\times10}+\cdots+\frac{1}{(3n-2)(3n+1)}=\frac{n}{3n+1}$ **Stel / Let $n=1$**: LK $=\frac{1}{4}$ RK $=\frac{1}{3+1}=\frac{1}{4}$ $\therefore$ LK = RK $\therefore$ bewering is waar vir / *statement is true for* $n=1$. **Aanvaar die bewering is waar as / *Accept the statement is true for* $n=k$** $\frac{1}{1\times4}+\frac{1}{4\times7}+\frac{1}{7\times10}+\cdots+\frac{1}{(3k-2)(3k+1)}=\frac{k}{3k+1}$ **Beskou / *Consider* $n=k+1$**: LK $=\frac{1}{1\times4}+\cdots+\frac{1}{(3k-2)(3k+1)}+\frac{1}{(3(k+1)-2)(3(k+1)+1)}$ $=\frac{k}{3k+1}+\frac{1}{(3k+1)(3k+4)}$ $=\frac{k(3k+4)+1}{(3k+1)(3k+4)}$ $=\frac{3k^2+4k+1}{(3k+1)(3k+4)}$ $=\frac{(k+1)(3k+1)}{(3k+1)(3k+4)}=\frac{k+1}{3k+4}$ RK $=\frac{k+1}{3(k+1)+1}=\frac{k+1}{3k+4}$ $\therefore$ LK = RK Die bewering is waar as $n=1$. As die bewering waar is vir $n=k$, is dit ook waar vir $n=k+1$. Dus is die bewering waar vir alle $n\in\mathbb{N}$. / *The statement is true for* $n=1$. *If the statement is true for* $n=k$, *it is also true for* $n=k+1$. *Thus, the statement is true for all* $n\in\mathbb{N}$.	✓ LK = RK $=\frac{1}{4}$ ✓ Rede ✓ Aanvaar... ✓ Stelling met $n=k$ ✓ Eerste k terme ✓ $(k+1)$de term ✓ Vervang eerste k terme met $\frac{k}{3k+1}$ ✓ Faktoriseer ✓ Vereenvoudig ✓ Antwoord van LK ✓ Antwoord van RK ✓ Storie **[12 punte / *marks*]**

Vraag / *Question* 3 **[19 punte / *marks*]**

NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
3.1 (a)	$(-2)^3 - 6(-2) - 4 = 0$ Dus $x + 2$ is 'n faktor van $x^3 - 6x - 4$	✓ $x = -2$ ✓ Vervang in vergelyking ✓ Antwoord $= 0$ **[3 punte / *marks*]**
3.1 (b)	$-2 \mid 1 \quad 0 \quad -6 \quad -4$ $\quad\quad 0 \quad -2 \quad 4 \quad 4$ $\quad\quad 1 \quad -2 \quad -2 \quad 0$ **OF** Langdeling kan ook gedoen word. $\therefore (x+2)(x^2 - 2x - 2) = 0$ $x = -2$ of $x = \frac{2 \pm \sqrt{(-2)^2 - 4(-2)}}{2}$ $x = 1 \pm \sqrt{3}$	✓ ✓✓ Elke term in $x^2 - 2x - 2$ ✓ $x = -2$ ✓ Vervang in formule ✓ $x = 1 \pm \sqrt{3}$ **[6 punte / *marks*]**
3.2	$x = 2 - 3i$ is 'n wortel, dus $x = 2 + 3i$ is ook. $(x - 2 + 3i)(x - 2 - 3i)$ **OF** $(x-2)^2 = (-3i)^2$ $= (x-2)^2 - 9i^2$ $x^2 - 4x + 4 = 9i^2$ $= x^2 - 4x + 13$ is 'n faktor $x^2 - 4x + 13 = 0$ Doen langdeling, dan is: $6x^4 - 37x^3 + 125x^2 - 149x - 65$ $= (x^2 - 4x + 13)(6x^2 - 13x - 5)$ $= (x^2 - 4x + 13)(2x - 5)(3x + 1) \in \mathbb{Z}[x]$	✓ Twee hakies ✓ Vermenigvuldig ✓ Faktor ✓ ✓✓✓ Langdeling $6x^2 - 13x - 5$ ✓ $(x^2 - 4x + 13)$ ✓ $(2x - 5)$ ✓ $(3x + 1)$ ***Indien verder ontbind, −1 punt.*** **[10 punte / *marks*]**

Vraag / *Question* 4 **[20 punte / *marks*]**

NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
4.1 (a)	$x = 7$	✓ ✓ Antwoord **[2 punte / *marks*]**
4.1 (b)	$x \in \mathbb{R}; x \neq 7$	✓ $x \in \mathbb{R}$ ✓ $x \neq 7$ **[2 punte / *marks*]**
4.1 (c)	Geen oplossing	✓ ✓ Antwoord **[2 punte / *marks*]**
4.1 (d)	$\frac{18}{\lvert x-5\rvert} \geq 6$ $18 \geq 6\lvert x-5\rvert$ $\lvert x-5\rvert \leq 3$ $-3 \leq x-5 \leq 3$ $2 \leq x \leq 8$	✓ Vermenigvuldig ✓ Vereenvoudig ✓ $-3 \leq x-5 \leq 3$ ✓ 2 ✓ 8 **[5 punte / *marks*]**
4.1 (e)	$2-\lvert x-8\rvert > 2$ $-\lvert x-8\rvert > 0$ $\lvert x-8\rvert < 0$ Geen oplossing	✓ Kry nul regs ✓ Vermenigvuldig met negatief ✓ Antwoord (Geen oplossing) **[3 punte / *marks*]**
4.2		✓ ✓ x-afsnitte $(-9; 0)$ en $(7; 0)$ ✓ y-afsnit $\left(0; \frac{7}{4}\right)$ ✓ ✓ Knakpunt $(-1; 2)$ ✓ Vorm **[6 punte / *marks*]**

Vraag / *Question* 5 **[15 punte / *marks*]**

NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
5.1	$C(-3;-2)$	✓✓ Antwoord **[2 punte / *marks*]**
5.2	x-afsnitte: $\|x+3\|-2=0$ $\|x+3\|=2$ $x+3=2$ of $x+3=-2$ $x=-1$ $x=-5$ y-afsnit: $y=\|0+3\|-2=1$ $\therefore A(-5;0)$ $B(-1;0)$ $D(0;1)$	✓ Vergelyking ✓ $x=-1$ ✓ $x=-5$ ✓ $y=1$ Indien nie koördinate as finale antwoord - maks 3/4. **[4 punte / *marks*]**
5.3	$\|x+3\|-2=\frac{3}{x-1}$ $\|x+3\|=\frac{3}{x-1}+2$ As $x\geq -3$: $x+3=\frac{3}{x-1}+2$ $(x+3)(x-1)=3+2(x-1)$ $x^2+2x-3=3+2x-2$ $x^2-4=0$ $(x-2)(x+2)=0$ $x=\pm 2$ $f(-2)=g(-2)=-1$ $f(2)=g(2)=3$ $E(2;3)$ $F(-2;-1)$	✓ Vergelyking ✓ Vergelyking $(x\geq -3)$ ✓ Vermenigvuldig met KGV ✓ Vereenvoudig ✓ $x=\pm 2$ ✓ $E(2;3)$ ✓ $F(-2;-1)$ **[7 punte / *marks*]**
5.4	$-5\leq x\leq -1$	✓ $-2\leq x$ ✓ $x\leq 0$ **[2 punte / *marks*]**

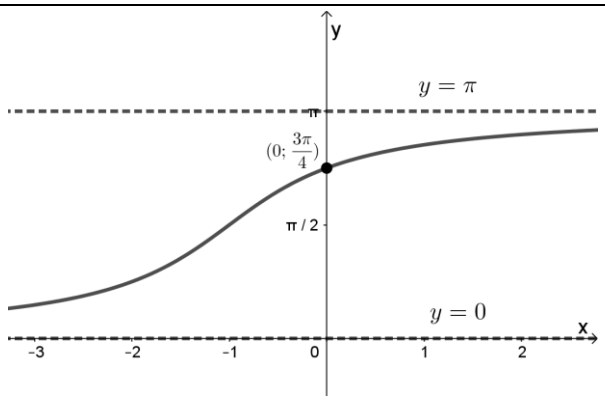
Vraag / *Question* 6 **[15 punte / *marks*]**

NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
6.1	$(2-5x)^{10} = \binom{10}{0}(2)^{10-0} + \binom{10}{1}(2)^9(-5x)^1 + \binom{10}{2}(2)^8(-5x)^2$ $= 1024 - 25600x + 288000x^2$	✓ ✓✓ Elke koëffisiënt ✓ Magte van x toenemend **[4 punte / *marks*]**
6.2 (a)	$\binom{12}{2}(6x^2)^{10}\left(-\frac{1}{3x}\right)^2$ $= 443418624x^{18}$	✓ ✓✓ Elke hakie ✓ Antwoord **[4 punte / *marks*]**
6.2 (b)	$\binom{12}{r}(6x^2)^{12-r}\left(-\frac{1}{3x}\right)^r$ $= \binom{12}{r}6^{12-r}\left(-\frac{1}{3}\right)^r x^{24-2r-r}$ $\therefore 24-2r-r=0 \implies r=8$ $= \binom{12}{8}6^4\left(-\frac{1}{3}\right)^8 x^0$ $= \frac{880}{9}$ **of** 97.78 **of** $97\frac{7}{9}$	✓ ✓✓ Elke hakie ✓ Vereenvoudig ✓ $r=4$ ✓ Vervang r in ✓ Antwoord **[7 punte / *marks*]**

Vraag / *Question* 7 **[17 punte / *marks*]**

NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
7.1	$\binom{6}{3}(3)^3(-2x)^3 = -4320x^3$ $\therefore b = -4320$ $\binom{6}{2}(3)^4(-2x)^2 = 4860x^2$ $\therefore a = 4860$ $\therefore \frac{a}{b} = -1.125$	✓ Metode ✓ $-4320x^2$ ✓ Metode ✓ $4860x^2$ ✓ $\frac{a}{b}$ **[5 punte / *marks*]**
7.2 (a)	$(1+x)(1-x)^{-4}$ $= (1+x)\left(1 + (-x)(-4) + \frac{(-4)(-5)}{2!}(-x)^2 + \frac{(-4)(-5)(-6)}{3!}(-x)^3 \dots\right)$ $= (1+x)(1+4x+10x^2+20x^3+\cdots)$ $= 1+5x+14x^2+30x^3+\cdots$	✓ Mag -4 ✓ ✓✓✓ Elke term uitbrei ✓ ✓ Vereenvoudig ✓ Vermenigvuldig **[8 punte / *marks*]**
7.2 (b)	$\frac{11}{9^4} = \frac{1+10}{(1-10)^4} = \frac{10(1+0.1)}{10^4(1-0.1)^4}$ $\therefore x = 0.1$ Vanuit vraag 7.2(a): $1+5(0.1)+14(0.1)^2+30(0.1)^3 = 1.67$ $x = 10$ is nie korrek nie!! x moet tussen $-1 < x < 1$ wees om te konvergeer na korrekte antwoord. Indien leerder $x = 10$ gebruik het, 0/4 punte.	✓ Faktoriseer 10 uit ✓ $x = 0.1$ ✓ Vervang $x = 0.1$ in ✓ Antwoord **[4 punte / *marks*]**

Vraag / *Question* 8 **[20 punte / *marks*]**

NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
8.1 (a)	$x = \text{bgtan}(y+1) + \frac{\pi}{2}$ $x - \frac{\pi}{2} = \text{bgtan}(y+1)$ $y + 1 = \tan\left(x - \frac{\pi}{2}\right)$ $y = \tan\left(x - \frac{\pi}{2}\right) - 1$ $\therefore h^{-1}(x) = \tan\left(x - \frac{\pi}{2}\right) - 1$	✓ Ruil x en y om ✓ Kry bgtan alleen ✓ Verander na tan ✓ Maak y die onderwerp **[4 punte / *marks*]**
8.1 (b)	$0 < x < \pi$ **OF** $x \in (0;\pi)$	✓ 0 ✓ π ✓ $<$ (slegs indien vorige 2 punte korrek is) **[3 punte / *marks*]**
8.1 (c)		✓ Asimptoot $y = \pi$ ✓ Asimptoot $y = 0$ ✓ y-afsnit $\left(0; \frac{3\pi}{4}\right)$ ✓ ✓ Vorm **[5 punte / *marks*]**
8.2 (a)	$g(x) = \frac{1}{2}\text{bgsin}\, x + \frac{\pi}{2}$ D_g: $-1 \le x \le 1$ $g(1) = \frac{1}{2}\text{bgsin}(1) + \frac{\pi}{2} = \frac{3\pi}{4}$ $\therefore$ A $\left(1; \frac{3\pi}{4}\right)$	✓ D_g ✓ Bepaal $g(1)$ ✓ $\frac{3\pi}{4}$ ✓ Antwoord **[4 punte / *marks*]**
8.2 (b)	f se definisieversameling is $-1 \le x \le 3$: $\therefore p = 2$ (translasie van 2 eenhede na regs vanaf $y = \text{bgcos}\, x$) $\therefore q = -\frac{\pi}{4}$ (translasie van $\frac{\pi}{4}$ eenhede afwaarts vanaf $y = \text{bgcos}\, x$) **OF** $\frac{3\pi}{4} = \text{bgcos}(1-2) + q$ $\therefore q = -\frac{\pi}{4}$	✓ ✓ p ✓ ✓ q **[4 punte / *marks*]**

Vraag / *Question* 9 **[18 punte / *marks*]**

NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
9.1	Area sektor ABC $= \frac{1}{2}\left(\sqrt{3}\right)^2\left(\frac{\pi}{6}\right) = \frac{\pi}{4}$	✓ Vervang in formule ✓ Antwoord i.t.v. π **[2 punte / *marks*]**
9.2	$\sin\frac{\pi}{3} = \frac{AB}{AO}$ $AO = \frac{\sqrt{3}}{\sin\frac{\pi}{3}} = 2$	✓ ✓ Vervang in sin-formule **[2 punte / *marks*]**
9.3	Area ΔAOB $= \frac{1}{2}(2)\left(\sqrt{3}\right)\sin\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ **OF** $OB^2 = 2^2 - \left(\sqrt{3}\right)^2$ (Pythagoras) $OB = 1$ Area ΔAOB $= \frac{1}{2}(1)\left(\sqrt{3}\right) = \frac{\sqrt{3}}{2}$	✓ Formule ✓ Vervang ✓ Antwoord in wortelvorm **[3 punte / *marks*]**
9.4	Area sektor COD $= \frac{1}{2}\left(2-\sqrt{3}\right)^2\left(\frac{\pi}{3}\right) = \frac{7-4\sqrt{3}}{6}\pi = 0.037592\ldots$ Area van gearseerde deel $= \frac{\sqrt{3}}{2} - \frac{\pi}{4} - 0.037592\ldots = 0.043$	✓ $2-\sqrt{3}$ ✓ Antwoord (Area van sektor) ✓ $\frac{\sqrt{3}}{2} - \frac{\pi}{4} - 0.037592\ldots$ ✓ Finale antwoord (3 des.) **[4 punte / *marks*]**
9.5	Boog CD $= \left(2-\sqrt{3}\right)\left(\frac{\pi}{3}\right) = 0.280595\ldots$ Boog BC $= \left(\sqrt{3}\right)\left(\frac{\pi}{6}\right) = 0.906899\ldots$ $OB^2 = 2^2 - \left(\sqrt{3}\right)^2$ (Pythagoras) $OB = 1$ ∴ DB $= 1 - \left(2-\sqrt{3}\right) = \sqrt{3} - 1$ Omtrek $= \sqrt{3} - 1 + 0.280595\ldots + 0.906899\ldots = 1.92$	✓ ✓ Vervang in formule (CD & BC) ✓ ✓ Antwoorde (CD & BC) ✓ ✓ DB ✓ Finale omtrek **[7 punte / *marks*]**

- EINDE VAN DIE MEMORANDUM / *END OF THE MEMORANDUM* -