α-MATHEMATICS

Alpha Mathematics

Half-year examination 2019

Grade 11 **Time: 2 ½ hours**

Total: 165 marks

INSTRUCTIONS AND INFORMATION

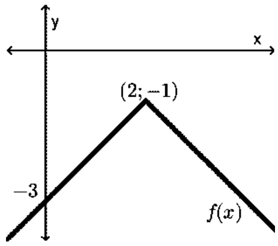
Carefully read through the following instructions before answering the question paper:

1. This question paper consists of 6 pages, a formula sheet of 1 page and an answer sheet of 1 page.
2. Answer ALL 9 questions.
3. Number the answers exactly the way the questions are numbered.
4. Non-programmable calculators may be used, unless otherwise indicated in the question.
5. Unless indicated otherwise, all answers, where necessary, must be given correct to two decimal places.
6. Cleary show all calculations, diagrams, graphs et cetera that you have used in determining the answers.
7. Answers only will not necessarily be awarded full marks.
8. The diagrams in the question paper are not necessarily drawn to scale.
9. All angles are given in radians. Answers must also be given in radians if necessary.
10. A formula sheet is included at the end of this question paper.
11. Write neat and legible.

Question 1 [20 marks]

This question must be answered **on the answer sheet**. Every question has **ONLY** one correct answer and is worth two (2) marks. Mark the correct answer with an **X** on the answer sheet.

1.1 The equation of $f(x)$ is:



(A) $y = |x-2| - 1$ **(B)** $y = |x-2| + 1$

(C) $y = |x+2| - 1$ **(D)** $y = |x+2| + 1$

1.2 $(2x-1)$ is a factor of:

(A) $2x^2 + 3x + 1$ **(B)** $2x^2 - x - 1$

(C) $4x^3 - 4x^2 - x + 1$ **(D)** $4x^3 - 3x - 1$

1.3 The expansion of $\left(\frac{1}{2} - x\right)^{-5}$ will converge if:

(A) $|x| < 1$ **(B)** $|x| < \frac{1}{2}$ **(C)** $|x| < 2$ **(D)** $|x| < \frac{1}{4}$

1.4 How many terms do the expansion of $\frac{(x^2-2x)^4}{10x}$ have?

(A) 3 **(B)** 4 **(C)** 5 **(D)** Infinity

1.5 If $|x| = 3x + 1$, then $x =$

(A) $-\frac{1}{2}$ **(B)** $-\frac{1}{4}$ **(C)** $-\frac{1}{2}$ of $-\frac{1}{4}$ **(D)** None x

1.6 The inverse function of $f(x) = \sin 4x$ will be defined if the domain of f is confined to $x \in \cdots$

(A) $\left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$ **(B)** $\left[-\frac{\pi}{4}; \frac{\pi}{4}\right]$ **(C)** $[-\pi; \pi]$ **(D)** $\left[-\frac{\pi}{8}; \frac{\pi}{8}\right]$

1.7 The term Ka^4b^{10} is a term in the expansion of $\left(\frac{1}{5}a + 10b^2\right)^9$, then $K = \cdots$

(A) 126 **(B)** 20000 **(C)** 20100 **(D)** 20160

1.8 Decompose into partial fraction: $\frac{g(x)}{x^2(x^2+1)} \equiv \cdots$, where $g(x)$ is any expression with real coefficients in terms of x.

(A) $\frac{A}{x^2} + \frac{B}{x^2+1}$ **(B)** $\frac{A}{x} + \frac{Bx+C}{x^2} + \frac{Dx+E}{x^2+1}$

(C) $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^2+1}$ **(D)** None of the previous answers.

1.9 The piecewise function of an absolute function is given:

$f(x) = \begin{cases} x-4 & \text{if} \quad x \le 5 \\ -x+6 & \text{if} \quad x > 5 \end{cases}$, then $f(0) + f(5) + f(10) =$

(A) -7 **(B)** 6 **(C)** -3 **(D)** 14

1.10 If a, b, c, d and e are real numbers and $a \neq 0$, then $ax^7 + bx^5 + cx^3 + dx + e = 0$

(A) has one real root. **(B)** at least one real root.

(C) has an odd number of non-real roots. **(D)** no real root.

Question 2 [21 marks]

2.1 Decompose $\frac{3x^2-2x+3}{(x^2+1)^2}$ into partial fractions. (9)

2.2 Use Mathematical induction and proof that the statement is valid for al $n \in \mathbb{N}$.

$$\frac{1}{1\times4} + \frac{1}{4\times7} + \frac{1}{7\times10} + \cdots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}$$ (12)

Question 3 [19 marks]

3.1 (a) Show that $(x+2)$ is a factor of $x^3 - 6x - 4$ by using the factor theorem. (3)

(b) Use synthetic division, or otherwise, and hence solve for $x \in \mathbb{R}$ in $x^3 - 6x - 4 = 0$. (6)

3.2 Factorize $6x^4 - 37x^3 + 125x^2 - 149x - 65$ completely over $\mathbb{Z}[x]$ if it is further given that $2 - 3i$ is a root. (10)

Question 4 [20 marks]

4.1 Solve for $x \in \mathbb{R}$ in:

(a) $|x-7| = 0$ (2)

(b) $|7-x| > 0$ (2)

(c) $|x+2| = -10$ (2)

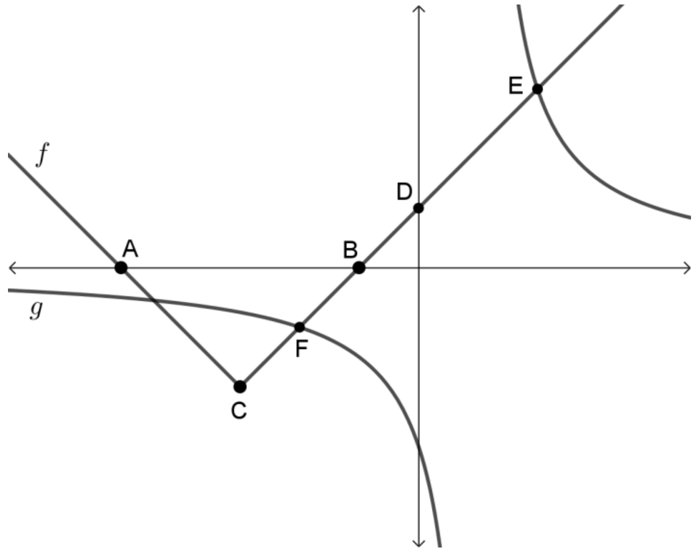
(d) $\frac{18}{|x-5|} \ge 6$ (5)

(e) $2 - |x-8| > 2$ (3)

4.2 Sketch the graph of $y = -\frac{1}{4}|x+1| + 2$. Clearly indicate all intercepts with the axes and the salient point on your sketch. (6)

Question 5 [15 marks]

Consider the sketch of $f(x) = |x + 3| - 2$ and $g(x) = \frac{3}{x-1}$. A, B and D are the intercepts of f with the axes and C is the salient point of f. E and F are the point of intersection of f and g.



5.1 Write down the coordinates of C, the salient point of f. (2)

5.2 Determine the coordinates of A, B and D, the intercepts of the f with the axes. (4)

5.3 Determine the coordinates of E and F, the point of intersection of f and g. (7)

5.4 Hence, give the value of x for which $-2 \leq f(x) \leq 0$. (2)

Question 6 [15 marks]

6.1 Determine and simplify the first 3 terms, in increasing powers of x, in the expansion of $(2 - 5x)^{10}$. (4)

6.2 Given: $\left(6x^2 - \frac{1}{3x}\right)^{12}$

(a) Determine the 3rd term in the expansion. (4)

(b) Determine the constant term in the expansion. (7)

Question 7 [17 marks]

7.1 The coefficients of x^2 and x^3 in the expansion of $(3 - 2x)^6$ is a and b respectively. Find the value of $\frac{a}{b}$ correct to 3 decimal places. (5)

7.2 (a) Determine and simplify the first four terms in the expansion of $\frac{1+x}{(1-x)^4}$. (8)

(b) Hence, use the results of question 7.2(a), and determine the value of $\frac{11}{9^4}$. (4)

Question 8 [20 marks]

8.1 Given: $h(x) = \arctan(x + 1) + \frac{\pi}{2}$

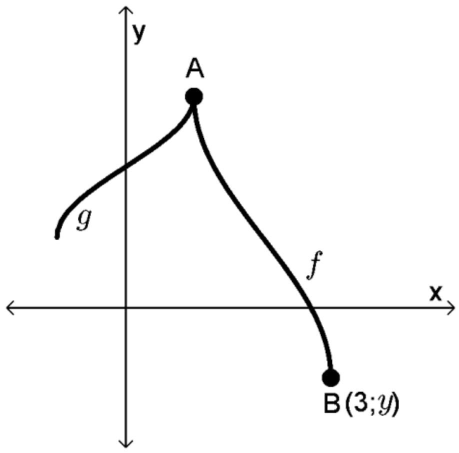
(a) Determine the inverse of $h(x)$ in the form $h^{-1}(x) = \cdots$ (4)

(b) $h(x)$ is a function, only if the domain of $h^{-1}(x)$ is restricted. Give the interval of the domain of $h^{-1}(x)$ so that $h(x)$ will be a function (3)

(c) Sketch $h(x)$ on an axes.

Clearly indicate the intercepts with the axes, the coordinates of the end points and asymptotes. (5)

8.2 Given the function $f(x) = \arccos(x - p) + q$ and $g(x) = \frac{1}{2}\arcsin x + \frac{\pi}{2}$. The functions have a common point of contact at A, one of the end points of f and g. B$(3; y)$ is an end point of f.

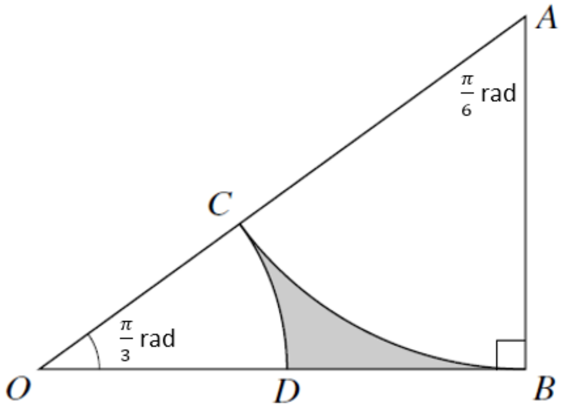


(a) Determine the coordinates of A. (4)

(b) Hence, or otherwise, determine the values of p and q. (4)

Question 9 [18 marks]

The diagram shows a triangle OAB where $A\hat{B}O$ is rectangular, $A\hat{O}B = \frac{\pi}{3}$ radians, $O\hat{A}B = \frac{\pi}{6}$ radians and $AB = \sqrt{3}$ units. Arc BC is part of a circle with center A and meets OA at C. Arc CD is part of a circle with center O and meets OB at D.



9.1 Determine the area of the sector ABC and leave your answer in terms of π. (2)

9.2 Show that $AO = 2$ units. (2)

9.3 Determine the area of ΔOAB and leave your answer in root form. (3)

9.4 Determine the area of the shaded region, correct to 3 decimal places. (4)

9.5 Determine the circumference of the shaded region. (7)

- END OF THE QUESTION PAPER -

ALPHA MATHEMATICS FORMULA SHEET

ALGEBRA:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$|x| = \begin{cases} x & \text{if} \quad x \geq 0 \\ -x & \text{if} \quad x < 0 \end{cases}$$

$$(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$$

$$(1+x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \cdots \quad ;\text{if } |x| < 1$$

VECTORS:

$$\boldsymbol{a} \bullet \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}| \cos\theta$$

$$\boldsymbol{a} \bullet \boldsymbol{b} = a_x b_x + a_y b_y + a_z b_z$$

CALCULUS:

$$\int_a^b x^n \, dx = \left[\frac{x^{n+1}}{n+1}\right]_a^b$$

$$V = \pi \int_a^b [f(x)]^2 \, dx$$

$$a_{n+1} = a_n - \frac{f(a_n)}{f'(a_n)}$$

TRIGONOMETRY:

In a sector: $s = r\theta$ and $A = \frac{1}{2} r^2 \theta$

Identities: $\sin^2 x + \cos^2 x = 1$ $\quad \tan^2 x + 1 = \sec^2 x$ $\quad \cot^2 x + 1 = \operatorname{cosec}^2 x$

$\operatorname{cosec} x = \dfrac{1}{\sin x}$ $\quad \sec x = \dfrac{1}{\cos x}$ $\quad \cot x = \dfrac{1}{\tan x}$

$y - y_1 = m(x - x_1)$ $\quad m = \dfrac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$

area ΔABC $= \dfrac{1}{2} ab.\sin C$

TABLE WITH DERIVATIVES:

$\boldsymbol{F(x)}$	$\boldsymbol{F'(x)}$
ax^n	nax^{n-1}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \,.\, \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \,.\, \cot x$

$\boldsymbol{F(x)}$	$\boldsymbol{F'(x)}$
bgsin x arcsin x	$\dfrac{1}{\sqrt{1-x^2}}$
bgcos x arccos x	$\dfrac{-1}{\sqrt{1-x^2}}$
bgtan x arctan x	$\dfrac{1}{x^2+1}$
$f(x).g(x)$	$f'(x).g(x) + f(x).g'(x)$
$\dfrac{f(x)}{g(x)}$	$\dfrac{f'(x).g(x) - f(x).g'(x)}{[g(x)]^2}$
$f[g(x)]$	$f'[g(x)].g'(x)$

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ANSWER SHEET

Name and Surname: ____

Question Total	1 [20]	2 [20]	3 [15]	4 [21]	5 [19]	6 [16]	7 [16]	8 [20]	9 [18]	TOTAL 165
Learner's mark										

Question 1

1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D