α-MATHEMATICS

Alpha Mathematics

Half-year examination 2019

Grade 12 **Time: 3 hours**

Total: 200 marks

INSTRUCTIONS AND INFORMATION

Carefully read through the following instructions before answering the question paper:

1. This question paper consists of 8 pages, a formula sheet of 3 pages and an answer sheet of 1 page.
2. Answer ALL 10 questions.
3. Number the answers exactly the way the questions are numbered.
4. Non-programmable calculators may be used, unless otherwise indicated in the question.
5. Unless indicated otherwise, all answers, where necessary, must be given correct to two decimal places.
6. Cleary show all calculations, diagrams, graphs et cetera that you have used in determining the answers.
7. Answers only will not necessarily be awarded full marks.
8. The diagrams in the question paper are not necessarily drawn to scale.
9. All angles are given in radians. Answers must also be given in radians if necessary.
10. A formula sheet is included at the end of this question paper.
11. Write neat and legible.

Question 1 [20 marks]

This question must be answered **on the answer sheet**. Every question has **ONLY** one correct answer and is worth two (2) marks. Mark the correct answer with an **X** on the answer sheet.

1.1 The equation of the tangent on $f(x) = e^{2x}$ at $x = 0$ is ...

(A) $y = 2x + 1$ **(B)** $y = 2x - 1$

(C) $y = -2x + 1$ **(D)** $y = -2x - 1$

1.2 For which values of x will the function $f(x) = x^2 + 2x - 15$ be increasing?

(A) $x \leq -5$ or $x \geq 3$ **(B)** $x \leq -1$

(C) $x \geq -1$ **(D)** $-5 \leq x \leq 3$

1.3 Given that $f(x) = \log_2(e^x)$, then $f'(x) =$

(A) $\frac{1}{\ln 2}$ **(B)** $\frac{1}{e^x \ln 2}$

(C) $\frac{1}{e^x}$ **(D)** $\frac{e^x}{\ln 2}$

1.4 Given $f(x) = 4e^{2x}$. Then the inverse of f in the form $f^{-1}(x) = \cdots$

(A) $\frac{1}{2} ln\left(\frac{x}{2}\right)$ **(B)** $ln\left(\frac{x}{4}\right)$

(C) $ln\left(\frac{x}{2}\right)$ **(D)** $\frac{1}{2} ln\left(\frac{x}{4}\right)$

1.5 For which values of a and b are the point $(1; 6)$ an inflection point of the function $f(x) = x^3 + ax^2 + bx + 1$?

(A) $a = 1$; $b = 1$ **(B)** $a = -3$; $b = 7$

(C) $a = -3$; $b = 5$ **(D)** $a = 1$; $b = -3$

1.6 For which value of $\boldsymbol{a}$ will the following matrix equation have no solution?

$$\begin{bmatrix} 2 & 3 \\ -1 & \boldsymbol{a} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 19 \\ 3 \end{bmatrix}$$

(A) $a = -\frac{3}{2}$ **(B)** $a = 3$

(C) $a = -\frac{2}{3}$ **(D)** $a = \frac{3}{2}$

1.7 Find the horisontal asymptote, if one exists, of $\frac{x^2+5x}{2x^3-x^2+4}$.

(A) $y = 1$ **(B)** $y = \frac{1}{2}$

(C) $y = 0$ **(D)** No horisontal asymptote.

1.8 $(1-i)^4 =$

(A) $\sqrt{2}e^{-\frac{\pi}{4}i}$ **(B)** $4e^{-\pi i}$

(C) $4e^{\pi i}$ **(D)** $\sqrt{2}e^{-\frac{\pi}{2}i}$

1.9 Given $y = f(x)$ is differentiable for all $x \in \mathbb{R}$. Further it is given that $f'(5) = 0$ and $f''(5) < 0$. Then $y = f(5)$ will have a ...

(A) local minimum. **(B)** local maximum.

(C) stationary point. **(D)** point of inflection.

1.10 If $ln\, f(x) = e^x\, ln\, x$, then $f'(1) = \cdots$

(A) $-\frac{1}{e}$ **(B)** $-e$

(C) $\frac{1}{e}$ **(D)** e

Question 2 [20 marks]

2.1 Solve for x and leave your answer in terms of e and ln, if necessary:

(a) $e^{2-x} = 6$ (3)

(b) $\frac{e^x}{e^x-1} = 9$ (3)

(c) $(ln\, x)^2 = ln\, e^2 x$ (6)

2.2 Consider the function $f(x) = ln(x-e) - 1$.

(a) Write down the equation of the vertical asymptote. (2)

(b) Make a sketch of $y = f(x)$ and show all intercepts and asymptotes clearly on your sketch. (6)

Question 3 [24 marks]

3.1 Anne-Marie researches the weight of female elephants living in captivity. By collecting data, they set the following formula up

$$G = 4000 - \frac{8000}{1.05 + 1.1^t}$$

with G being the weight in kilogram of a female elephant in captivity of t years.

(a) Determine the maximum weight of a female elephant in captivity. (2)

(b) Show, using mathematical calculations, that the weight of a female elephant in captivity is constantly increasing. (6)

(c) Calculate, using your calculations in question 3.1 (b), at what rate the weight of a 10-year-old female elephant increases in captivity. (2)

Patrick wants to write the formula of Anne-Marie so that he can quickly calculate the lifetime of a female elephant in captivity if he fills in the weight of the elephant. He makes the subject t in the given formula.

(d) Write the formula of G in the form $t = \log_d\left(\frac{a+bG}{c-G}\right)$ and calculate the age of a 3000 kg elephant. (6)

3.2 The following system of equations are given:

$$(\boldsymbol{b} - 1)x - 3y = 3 \text{ en } (\boldsymbol{b} + 3)x - 9y = 11$$

(a) Use Cramer's rule and determine the value of x in terms of $\boldsymbol{b}$. Show the matrices that you are using clearly. (6)

(b) Hence, determine the value of $\boldsymbol{b}$ if it is given that $x = 1$. (2)

Question 4 [16 marks]

4.1 Do the following calculations (in any form) and give the answer in exponential form. Use root form and π if necessary.

(a) $10\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) \times 5\left(\cos\frac{5\pi}{2} + i\sin\frac{5\pi}{2}\right)$ (3)

(b) $4\left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right) \div \left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$ (4)

4.2 Use De Moivre's theorem and show that the value of $\left(\frac{2i}{-1-\sqrt{3}i}\right)^{12}$ is real.
Work in polar form and use root form and π if necessary. (9)

Question 5 [24 marks]

Given: $(2x + y)^2 = 16x$

5.1 Show that $\frac{dy}{dx} = \frac{16-8x-4y}{4x+2y}$. (7)

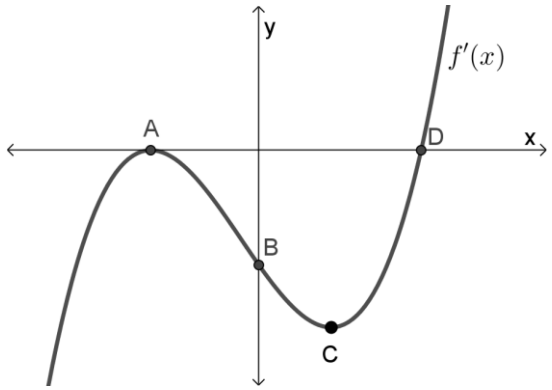
5.2 A **horisontal tangent** to the graph has a touch point at $(a; 2)$, with $a > 0$. Determine the value of a. (3)

5.3 Determine the nature of the stationary point at $(1; 2)$. (8)

5.4 Determine the equation of the normal line to the function where $x = 4$ and $y \geq 0$. (6)

Question 6 [22 marks]

Given the function $f'(x) = (x - 3)(x + 2)^2$, the **derivative** of $f(x)$.



6.1 Determine the coordinates of the points A, B, C and D. (9)

6.2 Determine and classify the x-coordinates of the stationery points of f. No calculations need to be shown. (5)

6.3 Write down the zeros/roots of f''. (2)

6.4 Determine the x-coordinate where f', the **derivative** of f, changes in concavity. (4)

6.5 Write down for which values of x is f decreasing? (2)

Question 7 [14 marks]

A function is given as $f(x) = \frac{2(x+1)}{(x+1)(x-3)^2} - 5x$.

7.1 Determine the value of the y-intercept. (2)

7.2 Determine the domain of f. (3)

7.3 Write down the equation(s), if any, of the:

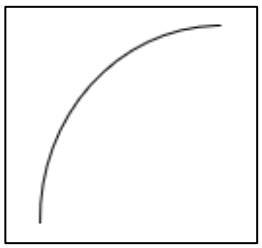
(a) horisontal asymptote. (1)

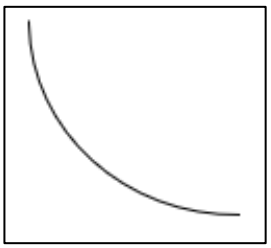
(b) vertical asymptote. (2)

(c) oblique asymptote. (2)

7.4 Determine the signs of the first and second derivatives of the following curves:

(a) (b)





(4)

Question 8 [18 marks]

Join the following rational function with the appropriate graphs, A – H, below:

8.1 $y = \frac{x^2-9}{x+3}$

8.2 $y = \frac{x^2+12x+27}{x-3}$

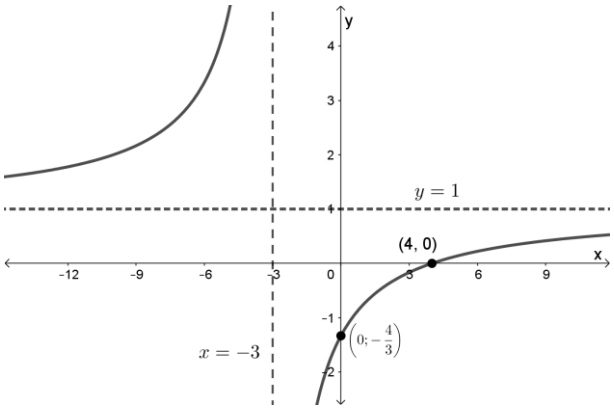
8.3 $y = \frac{2x^2-3x-9}{(x+3)^2}$

8.4 $y = \frac{2x^3-50x}{x^2-4}$

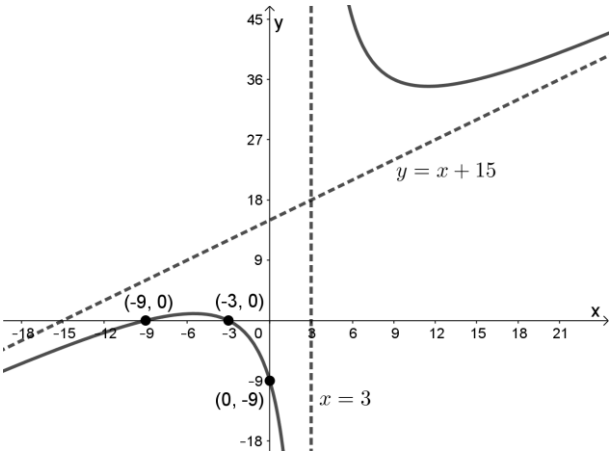
8.5 $y = \frac{-2x^2}{x^2-4}$

8.6 $y = \frac{x-4}{x+3}$ $(6 \times 3 = 18)$

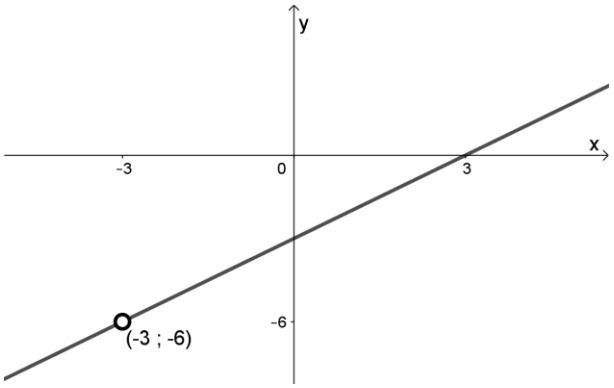
Graph A:



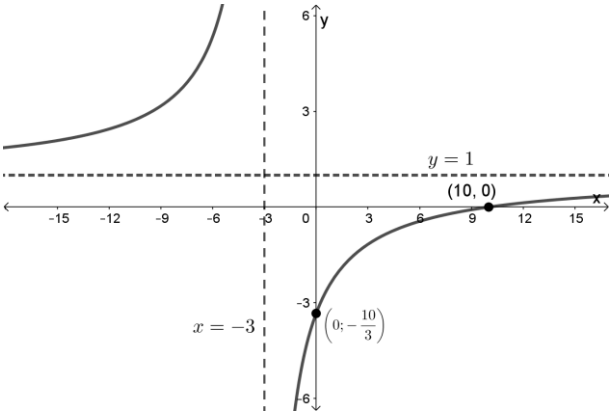
Graph B:



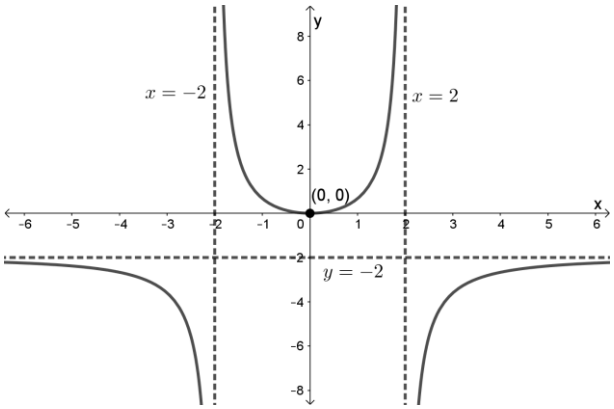
Graph C:



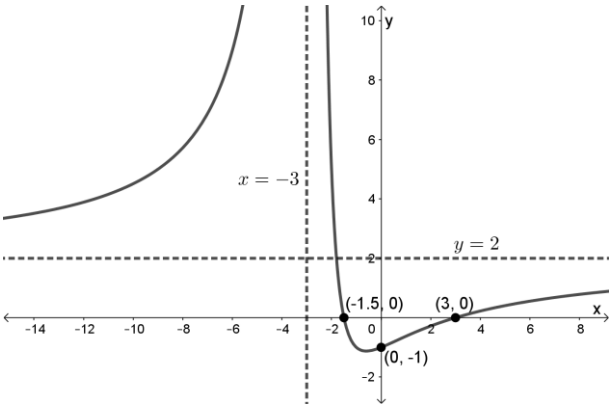
Graph D:



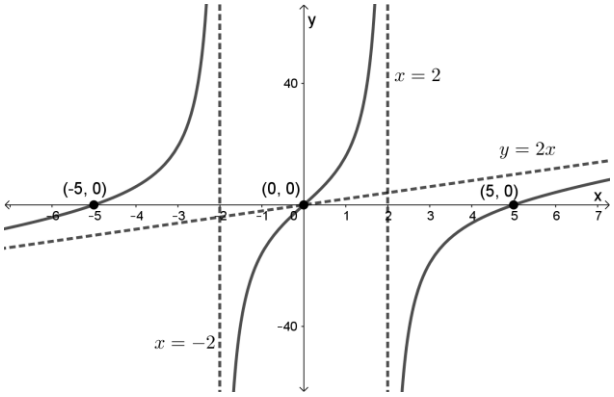
Graph E:



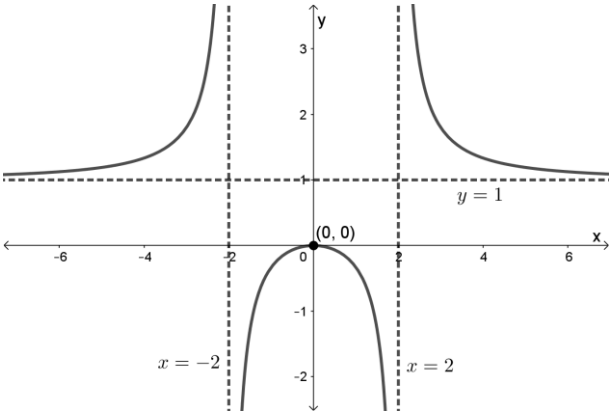
Graph F:



Graph G:



Graph H:



Question 9 [24 marks]

9.1 Die function $f(x)$ has the following properties:

- $f'(x) = 2^{4x+3}\ ln\ 2$
- The y-intercept of f is 3.

(a) Determine the function $f(x)$ that meets these requirements. Leave your answer in the form $f(x) = 2^{ax+b} + c$. (7)

(b) Show that the function $f(x)$ has no point of inflection. (4)

9.2 Determine the following integrals:

(a) $\int 2^{5x-7}\ dx$ (3)

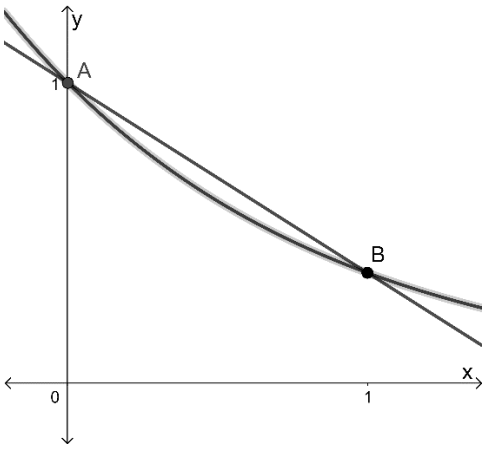
(b) $\int \left(\frac{5}{2x} + \frac{2x}{5}\right)\ dx$ (4)

(c) $\int \frac{2}{(x-3)\ ln\ 5}\ dx$ (2)

(d) $\int \frac{e^{3x} - 2e^{-3x}}{e^x}\ dx$ (4)

Question 10 [17 marks]

10.1 Given the function $f(x) = e^{-x}$. A and B are the points on the graph where $x = 0$ and $x = 1$. AB is connected to form a straight line.



(a) Show that the equation of the line through A and B is $y = (e^{-1} - 1)x + 1$. (3)

(b) Hence determine the area enclosed by the two graphs, between A and B. Leave your answer in the form $ae^{-1} + b$. (9)

10.2 Determine the volume of the solid that exists when the function $y^2(4x - 7) = 1$ rotates about the x-axis between $x = 2$ and $x = 3$. Leave your answer in the form $a\ ln\ b$. (6)

- END OF THE QUESTION PAPER -

Alpha Mathematics Grade 12 - Half-year examination 2019

ANSWER SHEET

Name and Surname: _____

Question Total	1 [20]	2 [20]	3 [24]	4 [16]	5 [24]	6 [22]	7 [14]	8 [18]	9 [24]	10 [18]
Learner's mark										

TOTAL 200

Question 1

1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D