α-WISKUNDE

Alpha Wiskunde Graad 12 / *Alpha Mathematics Grade 12*

Rekord eksamen 2019 / *Prelim examination 2019*

MEMORANDUM

Totaal / *Total*: 200 punte / *marks*

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Moderator: Anna Muller

Hierdie memorandum bestaan uit 13 bladsye. /
This memorandum consists of 13 pages.

Vraag / *Question* 1 [20 punte / *marks*]

1.1	A	B	C	**D**
1.2	A	**B**	C	D
1.3	**A**	B	C	D
1.4	A	B	**C**	D
1.5	A	**B**	C	D
1.6	A	B	**C**	D
1.7	A	B	C	**D**
1.8	A	B	C	**D**
1.9	A	B	**C**	D
1.10	A	**B**	C	D

Vraag / *Question* 2 **[20 punte / *marks*]**

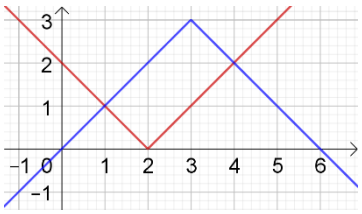
NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
2.1 (a)	$18 = ae^0 + 6$ $a = 12$	✔ Stel $t = 0$ ✔ Antwoord **[2 punte / *marks*]**
2.1 (b)	$T(2) = 12e^{-1} + 6$ = 10,41	✔ Stel $t = 2$ Antwoord **[2 punte / *marks*]**
2.1 (c)	$7 = 12e^{-\frac{t}{2}} + 6$ $\frac{1}{12} = e^{-\frac{t}{2}}$ $-\frac{t}{2} = ln\,(\frac{1}{12})$ $t = 5$ ure	✔ Vervang ✔ Vereenvoudig ✔ ln ✔ Antwoord, heelgetal **[4 punte / *marks*]**
2.1 (d)	6°C	✔ Antwoord **[1 punte / *marks*]**

NR. NO	**ANTWOORD / *ANSWER***	**PUNTE / *MARKS***
2.2	$x^2 - 2x - 4$ is 'n faktor Deur inspeksie / Langdeling: $(3x-5)(x^2-2x-4) = 0$ $x = \frac{5}{3}$	✔ x^2 ✔ $-2x$ ✔ -4 ✔ $3x-5$ ✔ Antwoord **[5 punte / *marks*]**
2.3	$(1+3x)^{-\frac{1}{3}}$ is: $1 + \left(-\frac{1}{3}\right)(3x) + \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)}{2!}(3x)^2$ $+ \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{7}{3}\right)}{3!}(3x)^3$ $= 1 - x + 2x^2 - \frac{14}{3}x^3$	✔ $-\frac{1}{3}$ ✔ $3x$ ✔ 1 ✔ $-x$ ✔ $2x^2$ ✔ $-\frac{14}{3}x^3$ **[6 punte / *marks*]**

Vraag / *Question* 3 **[16 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
3.1 (a)	$r = 1$ $\theta = \frac{5\pi}{4} / -\frac{3\pi}{4}$	✔ r ✔ θ **[2 punte / *marks*]**
3.1 (b)	$z - z^{-1}$ $= \cos\left(\frac{5\pi}{4}\right) + isin\left(\frac{5\pi}{4}\right) - \cos\left(-\frac{5\pi}{4}\right) - isin(-\frac{5\pi}{4})$ $= \cos\left(\frac{5\pi}{4}\right) + isin\left(\frac{5\pi}{4}\right) - \cos\left(\frac{5\pi}{4}\right) + isin(\frac{5\pi}{4})$ $= 2isin\left(\frac{5\pi}{4}\right)$	✔ Regte eksponent ✔ $\cos\left(-\frac{5\pi}{4}\right) - isin(-\frac{5\pi}{4})$ ✔ $-\cos\left(\frac{5\pi}{4}\right) + isin(\frac{5\pi}{4})$ ✔ Antwoord **[4 punte / *marks*]**
3.1 (c)	$-\sqrt{2}i$	✔ $-\sqrt{2}$ **[1 punte / *marks*]**
3.2	Stel $n = 1$: LK = 0; Rk=0. Dus waar vir $n = 1$. Aanvaar waar vir $n = k$: $(1 \times 0) + (2 \times 1) + (3 \times 2) + \cdots . + k(k-1) = \frac{k(k^2-1)}{3}$ Stel $n = k + 1$: LK = $\frac{k(k^2-1)}{3} + (k+1)k$ = $\frac{k^3-k+3k^2+3k}{3}$ RK = $\frac{(k+1)((k+1)^2-1)}{3}$ Beide dieselfde Bewering waar vir $n = 1$. As dit waar is vir $n = k$, dan is dit ook waar vir $n = k + 1$. Dus waar vir alle $n \in \mathbb{N}$.	✔ $n = 1$ ✔ Aanvaar $n = k$ ✔ $\frac{k(k^2-1)}{3}$ ✔ $+(k+1)k$ ✔ $\frac{k^3-k+3k^2+3k}{3}$ ✔ RK = $\frac{(k+1)((k+1)^2-1)}{3}$ ✔ ✓Beide dieselfde ✔ Storie **[9 punte / *marks*]**

Vraag / *Question* 4 **[21 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
4.1 (a)		✔ y afsnit ✔ ✓ ✓ x afsnitte ✔ Knakpunt ✔ Skets **[6 punte / *marks*]**
4.1 (b)	$1 \leq x \leq 4$	✔ Regte getalle ✔ Ongelykheid reg **[2 punte / *marks*]**
4.2	$\|A\| = \begin{vmatrix} a & 1 \\ 3 & 2a \end{vmatrix} = 2a^2 - 3$ $\|A_y\| = \begin{vmatrix} a & 8 \\ 3 & 15 \end{vmatrix} = 15a - 24$ Dus $\frac{15a-24}{2a^2-3} = 9$ $a = 1$	✓ ✓ $\|A\|$ ✓ ✓ $\|A_y\|$ ✓ Formule reg ✓Antwoord **[6 punte / *marks*]**
4.3	$y = 2bgcos\left(-\frac{1}{2}\right) - \frac{2\pi}{3} = \frac{2\pi}{3}$ $2bgcos\left(x - \frac{1}{2}\right) - \frac{2\pi}{3} = 0$ $x - \frac{1}{2} = \cos\left(\frac{\pi}{3}\right); x = 1$ $\left(-\frac{1}{2}; \frac{4\pi}{3}\right)$ en $\left(\frac{3}{2}; -\frac{2\pi}{3}\right)$	✓ y-afsnit ✓ ✓ x-afsnit ✓ ✓ $\left(-\frac{1}{2}; \frac{4\pi}{3}\right)$ ✓ ✓ $\left(\frac{3}{2}; -\frac{2\pi}{3}\right)$ **[7 punte / *marks*]**

Vraag / *Question* 5 **[19 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
5.1 (a)	$\|\boldsymbol{OP}\| = \sqrt{a^2 + b^2 + c^2}$ $\|\overline{\boldsymbol{a}}\| = \sqrt{29}$ $\left\|\overline{\boldsymbol{b}}\right\| = \sqrt{30}$	✔ Formule vir lengte ✔ $\|\overline{\boldsymbol{a}}\|$ ✔ $\left\|\overline{\boldsymbol{b}}\right\|$ **[2 punte / *marks*]**
5.1 (b)	$\boldsymbol{a} \bullet \boldsymbol{b} = \|\boldsymbol{a}\|\|\boldsymbol{b}\| cos\theta$ $cos\theta = \dfrac{-2 + 8 - 15}{\sqrt{29 \times 30}}$ $\theta = 1{,}88$	✔ Formule ✔ ✓ Vervang reg ✔ 1,88 **[4 punte / *marks*]**
5.1 (c)	$\|\overline{\boldsymbol{a}}\| - \left\|\overline{\boldsymbol{b}}\right\| = 3i - 2j - 8k$ $\beta = \underset{arccos}{\text{bgcos}}\left(\dfrac{u_2}{\|\boldsymbol{u}\|}\right) = \underset{arccos}{\text{bgcos}}\left(\dfrac{-2}{\sqrt{77}}\right)$ $\beta = 1{,}80$	✔ Aftrek ✔ -2 ✔ Antwoord **[3 punte / *marks*]**
5.2 (a)	$\begin{vmatrix} i & j & k \\ a & -2 & 3 \\ 2 & 4 & -3 \end{vmatrix}$ $= i(6 - 12) - j(-3a - 6) + k(4a + 4)$ $= -6i + (3a + 6)j + (4a + 4)k$	✔ Skryf in matriks vorm ✓ ✓ ✓ Haal uit matriks ✔ Antwoord **[5 punte / *marks*]**
5.2 (b)	$(\boldsymbol{u} \times \boldsymbol{v}).\boldsymbol{r} = 0$ $-90 + 15a + 30 = 0$ $a = 4$	✔ Formule ✓ ✓ Berekening ✔ Antwoord **[5 punte / *marks*]**

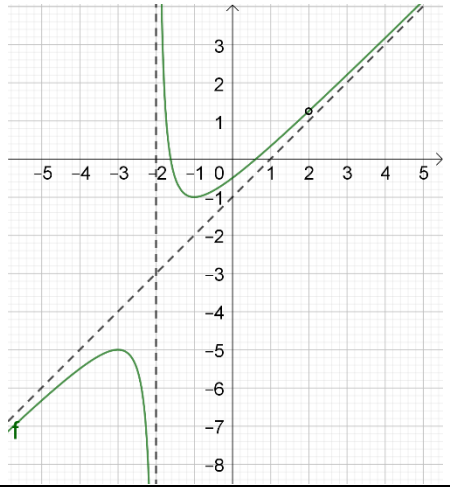
Vraag / *Question* 6 **[20 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
6.1 (a)	$f'(x) = 5^{2x}.ln5.ln(x^2).2 + 5^{2x}.\frac{2}{x}$ OF $5^{2x}.ln5.ln(x^2).2 + 5^{2x}.\frac{1}{x^2}.2x$	✓$5^x.ln5$ ✓2 ✓produkreël ✓✓ $\frac{2}{x}$ **[5 punte / *marks*]**
6.1 (b)	$\frac{1}{1+(e^x+e^{-x})^2}.(e^x - e^{-x})$	✔ Regte bgtan ✔ $e^x + e^{-x}$ in plek van x ✔ Kettingreël ✔ $-e^{-x}$ **[4 punte / *marks*]**
6.2 (a)	$\sqrt[3]{y} + \frac{1}{3}x.y^{-\frac{2}{3}}.y' - y'.\sqrt[3]{x} - y.\frac{1}{3}x^{-\frac{2}{3}} = 0$ $y' = \frac{\frac{1}{3}y.x^{-\frac{2}{3}} - \sqrt[3]{y}}{\frac{1}{3}x.y^{-\frac{2}{3}} - \sqrt[3]{x}}$	✓✓✓✓✓ elke stap (2e term 2 punte) ✓✓ bo en onder **[7 punte / *marks*]**
6.2 (b)	$m = \frac{\frac{1}{3}-1}{\frac{1}{3}-1} = 1$ $y - 1 = x - 1$ $y = x$	✓✓ Bereken en m=1 ✔ Formule ✔ Antwoord **[4 punte / *marks*]**

Vraag / *Question 7* **[22 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
7.1 (a)	$\frac{dy}{dx} = \frac{2cos2x(x+2) - sin2x}{(x+2)^2}$	✓✓bokant ✓ onderkant ✓kwosiënt reël **[4 punte / *marks*]**
7.1 (b)	$2cos2x.(x+2) = sin2x$ $tan2x = 2x + 4$	✔ $sin2x$ na regs ✔ Deel met $cos2x$ ✔ X 2 in hakie **[3 punte / *marks*]**
7.1 (c)	Stel $f(x) = tan2x - 2x - 4$ $f(0{,}6) < 0$ $f(0{,}7) > 0$	✔ $f(0{,}6) < 0$ ✔ $f(0{,}7) > 0$ **[2 punte / *marks*]**
7.1 (d)	$f'(x) = 2\sec^2 2x - 2$ $x_{n+1} = x_n - \frac{tan2x_n - 2x_n - 4}{2\sec^2 2x_n - 2}$ $\alpha = 0{,}6936$	✔ Afgeleide ✔ Newton ✔ Antwoord ✔ Reg afgerond **[4 punte / *marks*]**
7.2 (a)	$f'(x) = \frac{-3x^2}{7 - x^3}$ $f''(x) = \frac{-6x(7-x^3) + 3x^2(-3x^2)}{(7-x^3)^2} = 0$ $x(\ldots) = 0,\ \ x = 0$	✓✓ f' ✓✓ f'' ✓✓$x = 0$ **[6 punte / *marks*]**
7.2 (b)	$f''(-1) = \frac{39}{64} > 0$ $f''(1) = -\frac{5}{4} < 0$ Dis infleksie	✔ Kies 2 punte ✓✓ Bereken beide en verskil in teken. **[3 punte / *marks*]**

Vraag / *Question* 8 **[14 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
8.1	$f(2)$ bestaan nie	✓✓ rede en formaat **[2 punte / *marks*]**
8.2	$x = -2$ $y = x - 1$	✔ $x = -2$ ✓✓ $y = x - 1$ ✔ Deling **[4 punte / *marks*]**
8.3		✔ y afsnit ✔ x-afsnitte ✔ Maks draaipunt ✔ Minimum draaipunt ✔ Vert asimptoot ✔ Skuins asimptoot ✔ Oop kol $x = 2$ ✔ Algemene vorm **[8 punte / *marks*]**

Vraag / *Question* 9 **[35 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
9.1 (a)	$2\log_5 x + \int (co\sec^2 x - 1)dx$ $= 2\log_5 x - cotx - x + k$	✔ log ✔ Grondtal 5 ✔ Trig formule ✔ $cotx$ ✔ x **[5 punte / *marks*]**
9.1 (b)	Stel $u = \ln(5x)$ ✔ $du = \frac{1}{x}dx$✔ $\int u^5 du$✔ $= \frac{u^6}{6} + k$✔ Dus $\frac{(ln5x)^6}{6} + k$✔ Kan ook slegs antwoord gee.	**[5 punte / *marks*]**
9.1 (c)	$\int \left(\frac{1}{2}(1 + cos4x)\right)dx$ $= \frac{1}{2}\left(x + \frac{sin4x}{4}\right) + k$	✓ Formule ✓✓✓integraal **[4 punte / *marks*]**
9.2 (a)	$2x^2 - 5x + 8 = (Ax + B)x + C(x^2 + 4)$ $x = 0: 8 = 4C;\ \ C = 2$ $x^2: 2 = A + C;\ \ A = 0$ $x: -5 = B$ $\frac{2}{x} - \frac{5}{x^2 + 4}$	✓✓ A, B en C ✓✓✓ Waardes A, B en C ✓Antwoord **[6 punte / *marks*]**
9.2(b)	$2lnx - \int \frac{5dx}{4(\left(\frac{x}{2}\right)^2 + 1)}$ $= 2lnx - \frac{5}{4}bgtan\left(\frac{x}{2}\right).2 + k$	✓$\mathbf{2lnx}$ ✓$\mathbf{4}(\left(\frac{x}{2}\right)^2 + \mathbf{1})$ ✓ bgtan ✓✓$\frac{\mathbf{5}}{\mathbf{4}}$ en 2 **[5 punte / *marks*]**

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
9.3	$\Delta x_i = \frac{4-1}{n} = \frac{3}{n}$ $x_i = 1 + \frac{3}{n}i$ $f(x_i) = 2\left(1 + \frac{3}{n}i\right)^2 - 1$ $= 2 + \frac{12i}{n} + \frac{18i^2}{n^2} - 1$ $= 1 + \frac{12i}{n} + \frac{18i^2}{n^2}$ $f(x_i).\Delta x_i = \left(1 + \frac{12i}{n} + \frac{18i^2}{n^2}\right).\left(\frac{3}{n}\right) = \frac{3}{n} + \frac{36i}{n^2} + \frac{54i^2}{n^3}$ $\therefore \int_1^4 (2x^2 - 1)\ dx = \lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{3}{n} + \frac{36i}{n^2} + \frac{54i^2}{n^3}\right]$ $= \lim_{n\to\infty} \left[\frac{3}{n}\sum_{i=1}^{n} 1 + \frac{36}{n^2}\sum_{i=1}^{n} i + \frac{54}{n^3}\sum_{i=1}^{n} i^2\right]$ $= \lim_{n\to\infty} \left[\frac{3}{n} \times n + \frac{36}{n^2} \times \left(\frac{n^2}{2} + \frac{n}{2}\right) + \frac{54}{n^3} \times \left(\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}\right)\right]$ $= \lim_{n\to\infty} \left[3 + 18 + \frac{18}{n} + 18 + \frac{27}{n} + \frac{9}{n^2}\right] = 39$	✔ $\frac{3}{n}$ ✔ $1 + \frac{3}{n}i$ ✔ $f(x_i)$ ✔ Vereenvoudig $f(x_i)$ ✔ n ✔ $\frac{n^2}{2} + \frac{n}{2}$ ✔ $\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$ ✔ ✓ Vereenvoudig ✔ Antwoord **[10 punte / *marks***]

Vraag / *Question* 10 [13 punte / *marks*]

NR. NO	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
10.1	$f(x) = x \qquad g'(x) = e^x$ $f'(x) = 1 \qquad g(x) = e^x$ $\int x.e^x \ dx = xe^x - \int e^x \ dx$ $= xe^x - e^x + k$	✔ $f(x)$ ✓ $f'(x)$ ✔ $g(x)$ ✓ $g'(x)$ ✔ Vervang in formule ✔ Antwoord **[6 punte / *marks*]**
10.2	$A = \int_0^1 (x + 2 - xe^x) \ dx$ $= \frac{x^2}{2} + 2x - xe^x + e^x \Big\|_0^1$ $= \left(\frac{1}{2} + 2 - e + e\right) - (0 + 0 - 0 - 1)$ $= \frac{3}{2}$	✔ Bepaalde integraal ✔ Integreer ✔ Gebruik 10.1 ✔ Vervang grens in ✔ Vervang grens in ✔ $e^0 = 1$ ✔ Antwoord **[7 punte / *marks*]**

- EINDE VAN DIE MEMORANDUM / *END OF THE MEMORANDUM* -