α Mathematics

March 2023 **Time: 1 hour**

Grade 12 **Total: 70 marks**

INSTRUCTIONS AND INFORMATION

Carefully read through the following instructions before answering the question paper:

1. This question paper consists of two pages, a formula sheet of one page and an answer sheet of two pages.
2. Answer all 4 questions of the exam paper.
3. Number the answers in accordance to the questions.
4. Non-programmable calculators may be used, unless otherwise indicated at a specific question.
5. Unless otherwise mentioned, all applicable answers should be rounded off to **two decimal digits**.
6. **All necessary calculations, diagrams and graphs used to obtain answers must be shown clearly**.
7. The correct answer only, will not necessarily lead to full marks.
8. The diagrams in the question paper are not necessarily drawn on scale.
9. All angles are given in radians. Answers, where applicable, must be given in radians.
10. Present your work neatly and readable.

Question 1 [14 marks]

Answer this question on **the answer sheet**.

Each question has **ONLY ONE** correct answer and counts two (2) marks. Mark the correct answer with an **X** (cross) on the answer sheet.

1.1 Simplify $\frac{\left(2\text{cis}\frac{\pi}{4}\right)\left(4\text{cis}\frac{\pi}{2}\right)}{8\text{cis}\left(\frac{5\pi}{6}\right)}$

(A) $\frac{3}{4}\text{cis}\left(\frac{19\pi}{12}\right)$ **(B)** $\text{cis}\left(\frac{-\pi}{12}\right)$

(C) $\text{cis}\left(\frac{3\pi}{20}\right)$ **(D)** $\frac{3}{4}\text{cis}\left(\frac{3\pi}{20}\right)$

1.2 Change $p = 2e^{\frac{\pi}{3}i}$ to rectangular form.

(A) p cannot be changed **(B)** 2

(C) $1+\sqrt{3}i$ **(D)** $\sqrt{3}+i$

1.3 If $f(x) = \ln(\ln x)$, then $f'(x) =$

(A) $\frac{1}{x}$ **(B)** $\frac{1}{\ln x}$

(C) $\frac{\ln x}{x}$ **(D)** $\frac{1}{x\ln x}$

1.4 Determine $\frac{dy}{dx}$ if $\arctan y = x$

(A) $\frac{dy}{dx} = 1$ **(B)** $\frac{dy}{dx} = \frac{x}{1+y^2}$

(C) $\frac{dy}{dx} = \frac{1+y^2}{x}$ **(D)** $\frac{dy}{dx} = 1+y^2$

1.5 How many solutions will z have if $z^4 = -1+i$?

(A) 3 **(B)** 4

(C) 5 **(D)** No solutions

1.6 $\int \frac{dx}{x\ln 2} =$

(A) $\log_2 x + c$ **(B)** $\ln 2.\ln x + c$

(C) $\log_x 2 + c$ **(D)** No solution

1.7 If $y = \ln\left[\frac{3x}{e^{-x}}\right]$ determine $\frac{dy}{dx}$:

(A) $\frac{1}{x} + 1$ **(B)** $\ln 3 + x\ln e^{-x}$

(C) $\frac{3e^{-x} - 3xe^{-x}}{2^{-2x}}$ **(D)** $\frac{e^{-x}}{3x}$

Question 2 [18 marks]

2.1 The temperature, P, of bread, in degrees Celsius ($^{\circ}C$), t minutes after it is taken out of the oven, can be calculated with the formule $P(t) = 25 + 230e^{-0.1t}$.

(a) Calculate the temperature of the bread 5 minutes after taken out of the oven. (3)

(b) How long will it take the bread to cool down to $80^{\circ}C$? (3)

2.2 Consider $f(x) = y = \frac{\ln(4x-3)}{2}$

(a) Determine the inverse of f. Write your answer $f^{-1}(x) =$ (3)

(b) Show that f has **only** an x-intercept (and **not** a y-intercept) and hence calculate this x-intercept. (4)

(c) The function f has **one** asymptote. Determine the equation of the asymptote and state which type of asymptote it is. (2)

(d) Use the diagram sheet and draw the sketch of f. Show all intercepts and asymptotes clearly. . (3)

Question 3 [18 marks]

Determine the following:

3.1 $\frac{dy}{dx}$ if $y = 3^{\sin(e^{2x})}$ (5)

3.2 $f'(x)$ if $f(x) = [\log_4(\tan(3x))]^7$ (6)

3.3 $\int (2^{3x} + 3x^2)dx$ (4)

3.4 $\int \left[\ln 5 - \frac{1}{(-x-1)}\right] dx$ (3)

Question 4 [20 marks]

4.1 Given: $a = -1 + \sqrt{3}i$ and $b = 1 - i$

(a) Change a and b to polar form. (4)

(b) Use De Moivre's theorem and calculate b^4. Change your answer to rectangular form and show that b^4 is a real number. (4)

(c) Determine $\sqrt[3]{a}$. Give your answer in polar form. (5)

4.2 Use implicit differentiation and determine $\frac{dy}{dx}$ if $\ln(y+1) = e^y + x$ (7)

- END OF TEST-