

1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D
1.11	A	B	C	D
1.12	A	B	C	D
1.13	A	B	C	D
1.14	A	B	C	D
1.15	A	B	C	D

1.15 Verduideliking

**Verduideliking:**

$f(g(x)) = x$  Differentieer:

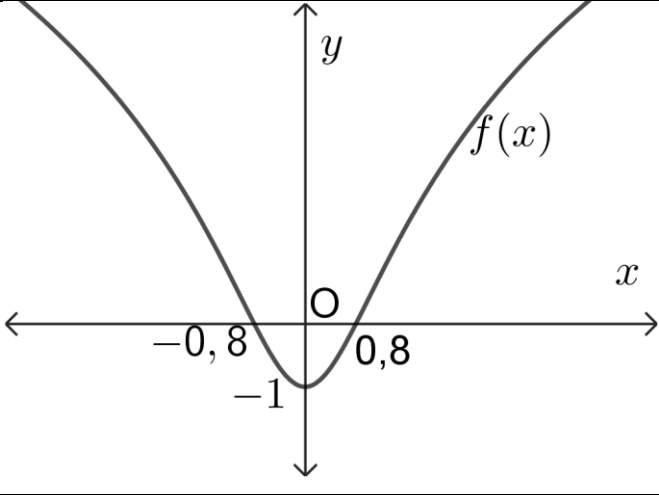
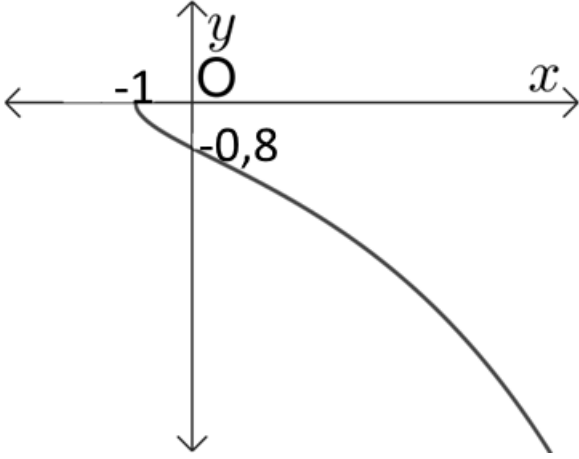
$$f'(g(x)) \times g'(x) = 1$$

Stel  $x = -2$ :

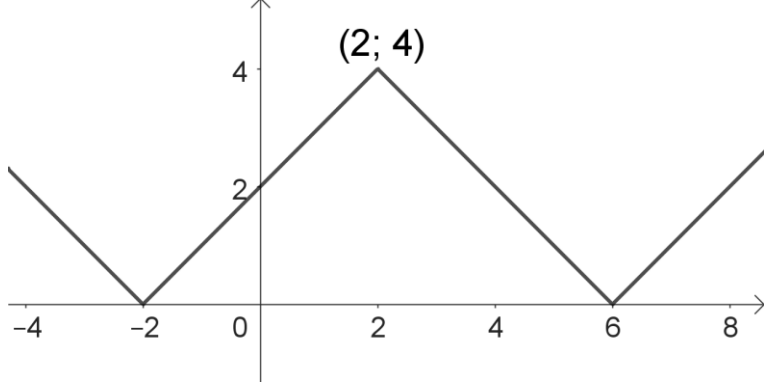
$$f'(g(-2)) \times g'(-2) = 1$$

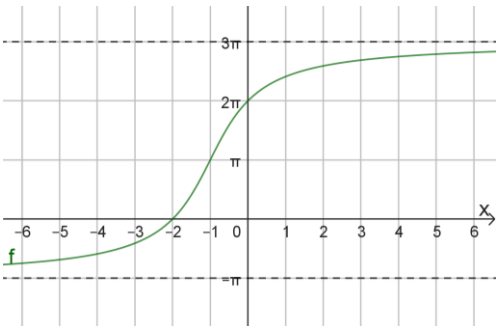
Dus  $f'(5) \times g'(-2) = 1$

$$\frac{1}{2} \times g'(-2) = 1, \text{ dus } g'(-2) = 2$$

2.1	$\ln(x^2 + 1) = \frac{1}{2}$ $x^2 + 1 = e^{\frac{1}{2}}$ $x = \pm\sqrt{e^{\frac{1}{2}} - 1}$ $x = \pm 0,8$	<ul style="list-style-type: none"> <li>✓ SV</li> <li>✓ Inverse</li> <li>✓ <math>x</math> alleen</li> <li>✓ Antwoord</li> </ul> <p style="text-align: right;">[4]</p>
2.2 (a)	$y = 2\ln 1 - 1$ $y = -1$	<ul style="list-style-type: none"> <li>✓ <math>x = 0</math></li> <li>✓ Antwoord en op grafiek</li> </ul> <p style="text-align: right;">[2]</p>
2.2 (b)		<ul style="list-style-type: none"> <li>✓ beide op grafiek</li> </ul> <p style="text-align: right;">[1]</p>
2.2 (c)	$x = 2\ln(y^2 + 1) - 1$ $\frac{x+1}{2} = \ln(y^2 + 1)$ $y^2 = e^{\frac{x+1}{2}} - 1$ $y = -\sqrt{e^{\frac{x+1}{2}} - 1}$	<ul style="list-style-type: none"> <li>✓ Ruil x en y</li> <li>✓ Vereenvoudig</li> <li>✓ Inverse</li> <li>✓ Antwoord</li> </ul> <p style="text-align: right;">[4]</p>
2.2 (d)	$x = -1 \text{ en } y = -0,8$	<ul style="list-style-type: none"> <li>✓✓ Antwoord</li> </ul> <p style="text-align: right;">[2]</p>
2.2 (e)		<ul style="list-style-type: none"> <li>✓ <math>x</math>-afsnit</li> <li>✓ <math>y</math>-afsnit</li> <li>✓ Vorm</li> </ul> <p style="text-align: right;">[3]</p>

<p>3.1</p>	$-1 - i = \sqrt{2} \operatorname{cis} \left( \frac{5\pi}{4} \right) \quad \text{ALT: } -1 - i = \sqrt{2} \operatorname{cis} \left( \frac{-3\pi}{4} \right)$ $(-1 - i)^4 = 4 \operatorname{cis} (5\pi) \quad (-1 - i)^4 = 4 \operatorname{cis} (-3\pi)$ $-\sqrt{3} + 1 = 2 \operatorname{cis} \left( \frac{5\pi}{6} \right) \quad -\sqrt{3} + 1 = 2 \operatorname{cis} \left( \frac{5\pi}{6} \right)$ $\frac{(-1-i)^4}{-\sqrt{3}+i} = 2 \operatorname{cis} \left( \frac{25\pi}{6} \right) \quad \frac{(-1-i)^4}{-\sqrt{3}+i} = 2 \operatorname{cis} \left( \frac{-23\pi}{6} \right)$	<p>✓✓ Poolvorm ✓ Tot 4e mag ✓✓ Poolvorm ✓ Antwoord</p> <p>[6]</p>
<p>3.2</p>	$x^3 = -1 = \operatorname{cis}(\pi + 2k\pi)$ $x^3 = 1 \operatorname{cis}(\pi) \text{ of } 1 \operatorname{cis}(3\pi) \text{ of } 1 \operatorname{cis}(5\pi)$ $x = 1 \operatorname{cis} \left( \frac{\pi}{3} \right) \text{ of } 1 \operatorname{cis}(\pi) \text{ of } 1 \operatorname{cis} \left( \frac{5\pi}{3} \right)$ $x = \frac{1}{2} + \frac{\sqrt{3}}{2} i \text{ of } x = -1 \text{ of } x = \frac{1}{2} - \frac{\sqrt{3}}{2} i$ <p style="text-align: center;">OF</p> $\text{Stel } z^3 = -1 = 1 \operatorname{cis}(\pi + 2k\pi)$ $\text{Dus } z = (1)^{\frac{1}{3}} \operatorname{cis} \left( \frac{\pi + 2k\pi}{3} \right), k = 0; 1; 2$ $k = 0: z = 1 \operatorname{cis} \left( \frac{\pi}{3} \right) = \frac{1}{2} + \frac{\sqrt{3}}{2} i$ $k = 1: z = 1 \operatorname{cis} \left( \frac{3\pi}{3} \right) = \operatorname{cis}(\pi) = -1$ $k = 2: z = 1 \operatorname{cis} \left( \frac{5\pi}{3} \right) = \frac{1}{2} - \frac{\sqrt{3}}{2} i$	<p>✓ poolvorm ✓ +2kπ ✓✓ 2 A in poolvorm ✓✓✓ Antwoorden in reghoekvorm</p> <p>[7]</p>
<p>3.3</p>	$x^2 = -1$ $(x^2 + 1) \text{ 'n faktor}$ $P(x) = x^4 - x^3 - 5x^2 - x - 6 = (x^2 + 1)(x^2 - x - 6)$ $P(x) = (x^2 + 1)(x - 3)(x + 2)$ $P(x) = (x - i)(x + i)(x - 3)(x + 2)$	<p>✓ -1 ✓ Faktor ✓✓ 2e Faktor ✓ Faktore</p> <p>[5]</p>

4.1 (a)	$(2; -4)$	✓ Antwoord [1]
4.1 (b)	$y =  0 - 2  - 4 = -2$ $ x - 2  = 4$ $x - 2 = \pm 4$ $x = 6$ of $x = -2$	✓ y-afsnit ✓ Stel $y = 0$ ✓ $\pm 4$ ✓ Antwoord [4]
4.1 (c)		✓✓ Afsnitte ✓ Knakpunt ✓ Vorm [4]
4.2	$\begin{vmatrix} 1 & 3 & 2 \\ 1 & -1 & -1 \\ 3 & 5 & 1 \end{vmatrix} = 1(-1 + 5) - 3(1 + 3) + 2(5 + 3) = 8$ Dus daar is 'n unieke oplossing.	✓ Regte matriks ✓ Bereken matriks ✓ Gevolgtrekking [3]
4.3	$(2x)^{9-r} \times \left(-\frac{5}{4x^2}\right)^r = ax^0$ $\therefore 9 - r - 2r = 0$ $r = 3$ $\binom{9}{3} (2x)^{9-3} \times \left(-\frac{5}{4x^2}\right)^3 = 84 \times -125 = -10\,500$	✓✓ Twee terme met binomiaal ✓ $r = 3$ ✓ Berekening ✓ Antwoord [5]

5.1 (a)	$AC = (2; 2; -1)$ $BC = (3; -2; 1)$	✓ AC ✓ BC [2]
5.1 (b)	$AC \cdot BC = 6 - 4 - 1 = 1$ $ AC  = \sqrt{4 + 4 + 1} = 3$ $ BC  = \sqrt{9 + 4 + 1} = \sqrt{14}$ $\cos \widehat{ACB} = (AC \cdot BC) /  AC  \times  BC $ $= \frac{1}{3\sqrt{14}}$ $\widehat{ACB} = 1,48 \text{ rad}$	✓ $AB \cdot BC$ ✓ $ AC $ ✓ $ BC $ ✓ Cos Ant ✓ Antwoord [5]
5.2	$3\vec{u} = 6\mathbf{i} - 3\mathbf{j} + 9\mathbf{k}$ $(3\vec{u}) \times \vec{v} = \begin{pmatrix} i & j & k \\ 6 & -3 & 9 \\ -1 & 5 & 1 \end{pmatrix}$ $= i(-3 - 45) - j(6 + 9) + k(30 - 3)$ $= (-48; -15; 27)$	✓ $3\vec{u}$ ✓✓✓ Kruisproduk [4]
5.3	$y = 4b \tan(1) + \pi = 2\pi$ $4b \tan(x + 1) + \pi = 0$ $x + 1 = \tan(-\frac{\pi}{4}); x = -2$ Asimptote: $y = 4 \times \frac{\pi}{2} + \pi = 3\pi$ en $y = 4 \times -\frac{\pi}{2} + \pi = -\pi$ 	✓ y-afsnit ✓ Bereken x-afsnit ✓ x-afsnit ✓ Asimptoot ✓ Asimptoot ✓ Vorm [6]
5.4(a)	$BD = 10$ Pyth $\tan \theta = \frac{10}{10\sqrt{3}}; \theta = \frac{\pi}{6}$	✓ BD ✓ $\tan \theta = \frac{10}{10\sqrt{3}}$ [2]

5.4 (b)	<p>Oppervl ABC = <math>\frac{1}{2} \times 300 \times \frac{\pi}{6} = 25\pi</math></p> <p>Oppervl <math>\Delta ABD = \frac{1}{2} \times 10\sqrt{3} \times 10 = 50\sqrt{3}</math></p> <p>Oppervl BDC = <math>50\sqrt{3} - 25\pi</math></p> <p><math>a = 50\sqrt{3}</math> en <math>b = -25</math></p>	<p>✓ Oppervl ABC</p> <p>✓ Oppervl <math>\Delta ABD</math></p> <p>✓ Oppervl BDC</p> <p>✓ <math>a</math> en <math>b</math></p> <p>[4]</p>
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## VRAAG 6

[18 punte]

6.1 (a)	<p><math>x = 9</math></p> <p>Want <math>f(9)</math> bestaan nie;</p> <p>Dus verwyderbare diskontinuiteit</p>	<p>✓ Punt</p> <p>✓ Rede</p> <p>✓ Tipe</p> <p>[3]</p>
6.1 (b)	<p><math>x = 2</math> en <math>x = 9</math></p>	<p>✓✓ Antwoorde</p> <p>[2]</p>
6.1 (c)	<p><math>x = 9</math>: want funksie is diskontinu daar.</p> <p><math>x = 2</math>: Rede: <math>\lim_{x \rightarrow 2^-} f'(x) = \lim_{x \rightarrow 2^-} (-2^x \ln 2) = -4 \ln 2</math></p> <p><math>\lim_{x \rightarrow 2^+} f'(x) = \lim_{x \rightarrow 2^+} \left( \frac{1}{(x-1) \ln 2} \right) = \frac{1}{\ln 2}</math></p> <p>Die afgeleide bestaan nie by <math>x = 2</math> nie.</p>	<p>✓ <math>x = 9</math> met rede</p> <p>✓✓ limiet van afgeleide links</p> <p>✓✓ limiet van afgeleide regs</p> <p>-1 indien notasie fout</p> <p>[5]</p>
6.2	<p>Stel <math>n = 1</math>: Lk = <math>a</math> en RK = <math>a</math>. Dus waar vir <math>n = 1</math>.</p> <p>Aanvaar bewering waar vir <math>n = k</math>:</p> <p><math>a + ar + ar^2 + \dots + ar^{k-1} = \frac{a(r^k - 1)}{r - 1}</math></p> <p>Stel <math>n = k + 1</math>:</p> <p>LK = <math>\frac{a(r^k - 1)}{r - 1} + ar^k</math></p> <p><math>= \frac{ar^k - a + ar^k(r - 1)}{r - 1}</math></p> <p><math>= \frac{ar^k - a + ar^{k+1} - ar^k}{r - 1}</math></p> <p><math>= \frac{ar^{k+1} - a}{r - 1}</math></p> <p><math>= \frac{a(r^{k+1} - 1)}{r - 1} = \text{RK}</math> Dus bewering waar vir <math>n = k + 1</math></p> <p>Volgens die beginsel van wiskundige induksie, is die bewering dus waar vir alle <math>n \in \mathbb{N}</math></p>	<p>✓ <math>n = 1</math></p> <p>✓ Aanvaar waar</p> <p>✓✓ <math>n = k + 1</math>, LK 1e stap</p> <p>✓ Dieselfde noemer</p> <p>✓ Vereenvoudig</p> <p>✓ RK</p> <p>✓ Rede</p> <p>[8]</p>

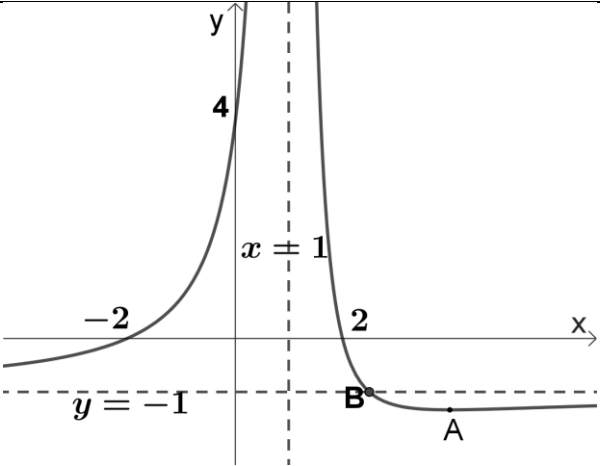
7.1 (a)	$e^{x^2+\cos x} \times (2x - \sin x)$	<ul style="list-style-type: none"> <li>✓ <math>e</math></li> <li>✓ <math>2x</math></li> <li>✓ <math>-\sin x</math></li> </ul> <p style="text-align: right;">[3]</p>
7.1 (b)	$g(x) = \ln(x - 5)^2$ $= 2\ln(x - 5)$ $g'(x) = \frac{2}{x-5}$ <p style="text-align: center;">OF</p> $g'(x) = \frac{2x-10}{x^2-10x+25}$ $= \frac{2(x-5)}{(x-5)^2} = \frac{2}{x-5}$	<ul style="list-style-type: none"> <li>✓ Faktoriseer</li> <li>✓ Log wet</li> <li>✓ Antwoord</li>   <li>✓ Diff</li> <li>✓ Faktoriseer</li> <li>✓ Antwoord</li> </ul> <p style="text-align: right;">[3]</p>
7.2	$3y^2 \cdot y' - y - xy' = y' + 2x$ $y' = \frac{2x+y}{3y^2-x-1}$ $m = \frac{1}{1} = 1$ $y + 1 = x - 1 \quad \therefore y = x - 2$	<ul style="list-style-type: none"> <li>✓✓✓ Diff.</li> <li>✓ <math>y' =</math></li> <li>✓ <math>m = 1</math></li> <li>✓ Vergelyking</li> </ul> <p style="text-align: right;">[6]</p>
7.3 (a)	$f(1) = 1^2 - \sqrt{4+1} = -1,24 < 0$ $f(2) = 2^2 - \sqrt{4+2} = 1,55 > 0$ <p>Maar f is kontinu op die interval, dus f het hier 'n nulpunt.</p>	<ul style="list-style-type: none"> <li>✓ <math>f(1)</math></li> <li>✓ <math>f(2)</math></li> </ul> <p style="text-align: right;">[2]</p>
7.3 (b)	$f'(x) = 2x - \frac{1}{2}(4+x)^{-\frac{1}{2}}$ $a_{n+1} = a_n - \frac{a_n^2 - \sqrt{4+a_n}}{2a_n - \frac{1}{2}(4+a_n)^{-\frac{1}{2}}}$ $a_1 = 1,592$ $a_2 = 1,534885$ $a_3 = 1,533751$ $a_4 = 1,533751$ $x \approx 1,53375$	<ul style="list-style-type: none"> <li>✓ Differensieer</li>   <li>✓✓ Formule</li>   <li>✓ <math>a_1 = 1,592</math></li>   <li>✓ Antwoord</li> <li>✓ Reg afgerond</li> </ul> <p style="text-align: right;">[6]</p>

<p>8.1</p>	$\Delta x_i = \frac{3}{n}; x_i = \frac{3}{n} \cdot i$ $f(x_i) = -\left(\frac{3}{n} \cdot i\right)^2 + 6 \cdot \frac{3}{n} \cdot i = -\frac{9}{n^2} i^2 + \frac{18i}{n}$ $\sum_{i=1}^n \left(-\frac{9}{n^2} i^2 + \frac{18i}{n}\right)$ $= -\frac{9}{n^2} \left(\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}\right) + \frac{18}{n} \left(\frac{n^2}{2} + \frac{n}{2}\right)$ $= -3n - \frac{9}{2} - \frac{3}{2n} + 9n + 9$ $= 6n + \frac{9}{2} - \frac{3}{2n}$ <p>basis x hoogte</p> $= \frac{3}{n} \times \left(6n + \frac{9}{2} - \frac{3}{2n}\right) = 18 + \frac{27}{2n} - \frac{9}{2n^2}$ $\int_0^3 (-x^2 + 6x) dx = \lim_{n \rightarrow \infty} \left(18 + \frac{27}{2n} - \frac{9}{2n^2}\right)$ $= 18$	<p>✓ <math>\Delta x_i</math></p> <p>✓ <math>x_i</math></p> <p>✓ <math>f(x_i)</math></p> <p>✓ sigma <math>i^2</math></p> <p>✓ sigma <math>i</math></p> <p>✓ Vereenvoudig</p> <p>✓ b x h</p> <p>✓ lim</p> <p>✓ Antwoord</p> <p style="text-align: right;">(9)</p>
<p>8.2 (a)</p>	$f(x) = \int (4x^3 + 6x^2 - 2) dx$ $= x^4 + 2x^3 - 2x + k$ <p>(0; -1): <math>-1 = (0)^4 - 2(0)^3 - 2(0) + k</math></p> $k = -1$ $f(x) = x^4 + 2x^3 - 2x - 1$	<p>✓✓ Integreer</p> <p>y-afsnit word gegee</p> <p>✓ <math>k = 3</math></p> $f(x) = x^4 + 2x^3 - 2x + k$ <p>Voldoende v 2/3</p> <p style="text-align: right;">[3]</p>
<p>8.2 (b)</p>	$f'(x) = 2(x+1)^2(2x-1) = 0$ $x = -1 \text{ of } x = \frac{1}{2}$	<p>✓ x- waardes 1e afgeleide</p> <p style="text-align: right;">[1]</p>
<p>8.2 (c)</p>	$f''(x) = 12x^2 + 12x = 0$ $x = 0 \text{ of } x = -1$	<p>✓ Bepaal <math>f''</math></p> <p>✓ x- waardes 2e afgeleide</p> <p style="text-align: right;">[2]</p>
<p>8.2 (d)</p>	$x = \frac{1}{2}$	<p>✓ Antwoord</p> <p style="text-align: right;">[1]</p>
<p>8.2 (e)</p>	<p><math>x = -1</math>:</p> $f''(-2) = 24 > 0$ $f''\left(-\frac{1}{2}\right) = -3 < 0$ <p>Dus buigpunt want teken van <math>f''(a)</math> verkil weerskante van <math>x = -1</math></p>	<p>✓ Antwoord</p> <p>✓✓ Motivering</p> <p style="text-align: right;">[3]</p>

## VRAAG 9

[19 punte]

9.1	$\frac{\tan(4x+1)}{4} - \frac{\operatorname{cosec} 3x}{3} + \pi x + k$	<ul style="list-style-type: none"> <li>✓ tan</li> <li>✓ ÷ 4</li> <li>✓ cot</li> <li>✓ ÷ 3</li> <li>✓ <math>\pi x + k</math></li> </ul> <p style="text-align: right;">(5)</p>
9.2	<p>Stel <math>\frac{8x^2-x-2}{(4x^2+1)(x+4)} = \frac{Ax+B}{4x^2+1} + \frac{C}{x+4}</math></p> <p><math>8x^2 - x - 2 = (Ax + B)(x + 4) + C(4x^2 + 1)</math></p> <p>Stel <math>x = -4</math>: <math>130 = C(65)</math>; <math>C = 2</math></p> <p><math>x^2</math>: <math>A + 4C = 8</math>; <math>A = 0</math></p> <p><math>x</math>: <math>4A + B = -1</math> of <math>4B + C = -2</math>; <math>B = -1</math></p> <p><math>\int \left( -\frac{1}{(2x)^2+1} + \frac{2}{x+4} \right) dx</math></p> <p><math>= \frac{\operatorname{bgtan}(2x)}{2} + 2 \ln x + 4  + k</math></p>	<ul style="list-style-type: none"> <li>✓ Pars breuke</li> <li>✓ Maal kgv</li> <li>✓✓✓ A, B en C</li> <li>✓✓ bgtan</li> <li>✓ ln</li> </ul> <p style="text-align: right;">[8]</p>
9.3	<p>Stel <math>f(x) = x</math>; <math>f'(x) = 1</math></p> <p><math>g'(x) = \cos^2 x = \frac{1}{2}[1 + \cos(2x)]</math></p> <p><math>\therefore g(x) = \frac{1}{2}x + \frac{1}{4}\sin(2x)</math></p> <p><math>\int (x \cdot \cos^2(x)) dx</math></p> <p><math>= x \left( \frac{1}{2}x + \frac{1}{4}\sin 2x \right) - \int \left( \frac{1}{2}x + \frac{1}{4}\sin 2x \right) dx</math></p> <p><math>= x \left( \frac{1}{2}x + \frac{1}{4}\sin 2x \right) - \frac{1}{4}x^2 + \frac{1}{8}\cos 2x + k</math></p>	<ul style="list-style-type: none"> <li>✓ <math>f(x)</math></li> <li>✓ <math>g'(x)</math></li> <li>✓ Identiteit</li> <li>✓ Integreer</li> <li>✓ Pas formule toe</li> <li>✓ Integreer</li> </ul> <p style="text-align: right;">[6]</p>

<p>10.1 (a)</p>	$f'(x) = \frac{-2x(x-1)^2 - 2(4-x^2)(x-1)}{(x-1)^4} = 0$ $(x-1)(-2x^2 + 2x - 8 + 2x^2) = 0$ $x \neq 1, \text{ dus } 2x - 8 = 0 \quad \text{ALT: } 2x^2 - 10x + 8 = 0$ $x = 4 \quad \quad \quad \therefore x^2 - 5x + 4 = (x-4)(x-1) = 0$ $y = -\frac{4}{3} \quad \quad \quad \therefore x = 4$ <p>OF Vereenvoudig: <math>2x^2 - 10x + 8 = 0</math>  <math>(x-1)(x-4) = 0</math></p>	<p>✓✓ <math>f'(x) = 0</math>                  ✓ Vereenvoudig                  ✓ <math>x = 4</math>                  ✓ <math>y = -\frac{4}{3}</math></p> <p>[5]</p>
<p>10.1 (b)</p>	$\frac{4-x^2}{(x-1)^2} = -1$ $4 - x^2 = -x^2 + 2x - 1$ $x = \frac{5}{2}$	<p>✓ Stel vgl = -1                  ✓ Vereenvoudig                  ✓ Antwoord</p> <p>[3]</p>
<p>10.1 (c)</p>	<p><math>x = 1</math></p>	<p>✓ Asimptoot</p> <p>(1)</p>
<p>10.1 (d)</p>		<p>✓ Asimptote                  ✓ Punt B</p> <p>(2)</p>
<p>10.1 (e)</p>	$V = \pi \int_2^4 \frac{(4-x^2)^2}{(x-1)^4} dx \quad \text{OF} \quad V = \pi \int_2^4 \left( \frac{4-x^2}{(x-1)^2} \right)^2 dx$	<p>✓✓ Formule reg</p> <p>(2)</p>
<p>10.2</p>	$\int_1^e \frac{a+\ln x}{x} = 3,5 \quad \text{OF} \quad \int_1^e \left( \frac{a}{x} + \frac{\ln x}{x} \right) dx$ $\left. \frac{(a+\ln x)^2 e}{2} \right _1 = 3,5 \quad \text{OF} \quad \left. (a \ln x + \frac{1}{2} (\ln x)^2) e \right _1 = 3,5$ $(a + \ln e)^2 - (a + \ln 1)^2 = 7 \quad \text{OF} \quad a(1) + \frac{1}{2}(1) = 3,5$ $(a+1)^2 - a^2 = 7$ $a^2 + 2a + 1 - a^2 = 7$ $2a = 6$ $a = 3$	<p>✓ Integraal                  ✓ Stel = 3,5                  ✓✓ Integreer                  ✓ Vervang                  ✓ Vereenvoudig</p> <p>✓ Antwoord</p> <p>(7)</p>