

α MATHEMATICS

June 2025

Grade 12

Time: 3 hours

Total: 200 marks

Instructions and information

Read the following instructions carefully before answering the question paper:

1. This question paper consists of seven pages, a three-page answer sheet and a three-page formula sheet .
2. Answer ALL twelve questions.
3. Number the answers as the questions are numbered.
4. Non- programmable calculators may be used, unless otherwise stated in a question.
5. Unless otherwise specified, all answers, where applicable, must be rounded correctly to **two decimal places** .
6. **indicate all necessary calculations**, diagrams, graphs, etc. that you used to determine your answers.
7. Full marks will not necessarily be awarded to answers only.
8. The diagrams in the question paper are not necessarily drawn to scale.
9. All angles are given in radians. Answers should be given in radians if necessary.
10. Write neatly and legibly.

Question 1**[30 marks]**

- Answer this question **on the answer sheet** by placing an X (cross) on A, B, C or D.
- These questions count 2 marks each.

1.1 When will the expansion of $\sqrt{2-x}$ converge?

- (A) $|x| > \frac{1}{2}$ (B) $|x| > 2$
(C) $|x| < \frac{1}{2}$ (D) $|x| < 2$

1.2 Which of the following will be a tangent of $y = \frac{1}{\sqrt{x}}$ at $x = 1$?

- (A) $y = -x + \frac{1}{2}$ (B) $y = -\frac{x}{2} + \frac{3}{2}$
(C) $y = x + \frac{1}{2}$ (D) $y = -x + 2$

1.3 Solve for x : $\left|\frac{x}{2} + 3\right| < 1$

- (A) $-8 < x < -4$ (B) $x < -8$ or $x > -4$
(C) $-4 < x < 8$ (D) $x < -4$ or $x > 8$

1.4 Determine the salient point and the y -intercept of $y = |x - 1| - 2$.

- (A) salient point: $(-1; 1)$; y -intercept : $y = -1$
(B) salient point: $(-1; -1)$; y -intercept : $y = 1$
(C) salient point: $(1; -2)$; y -intercept : $y = -1$
(D) salient point: $(1; -2)$; y -intercept : $y = 1$

1.5 If $z^2 = \cos \frac{\pi}{5} + i \sin \frac{\pi}{5} = \text{cis} \frac{\pi}{5}$, determine the value of z .

- (A) $z = \text{cis} \left(\frac{\pi}{10}\right)$ (B) $z = \text{cis} \left(\frac{\pi}{10}\right)$ or $z = \text{cis} \left(\frac{11\pi}{10}\right)$
(C) $z = \text{cis} \left(\frac{11\pi}{10}\right)$ (D) None of the above

1.6 The asymptote of $y = e^{x+2} - 1$ is

- (A) $y = -1$ (B) $y = 1$
(C) $x = -2$ (D) $x = 2$

1.7 The quotient of $2\text{cis}\left(\frac{\pi}{3}\right)$ and $4\text{cis}\left(\frac{\pi}{6}\right)$ is

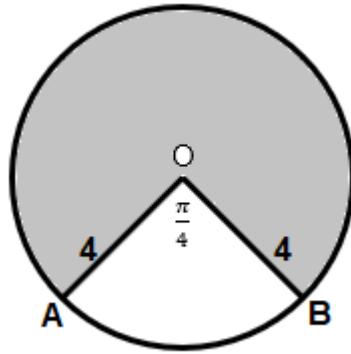
(A) $8\text{cis}\left(\frac{\pi}{2}\right)$

(B) $2\text{cis}\left(\frac{\pi}{6}\right)$

(C) $6\text{cis}\left(\frac{\pi}{2}\right)$

(D) $\frac{1}{2}\text{cis}\left(\frac{\pi}{6}\right)$

1.8 In the sketch below, O is the centre of a circle with radius 4cm.
 $\widehat{AOB} = \frac{\pi}{4}$ radians.



Determine the area of the shaded area.

(A) $2\pi \text{ cm}^2$

(B) $\frac{\pi}{2} \text{ cm}^2$

(C) $14\pi \text{ cm}^2$

(D) $\frac{7\pi}{2} \text{ cm}^2$

1.9 If $f(x) = 2x^3 - 7x^2 + x + 1$, which of the following, according to the rational root theorem, are possible zeros of f ?

(A) ± 1

(B) ± 2

(C) ± 1 and ± 2

(D) ± 1 and $\pm \frac{1}{2}$

1.10 If $x = -1 - 2i$ is a root of the equation $p(x) = 0$, which one of the following is a root of $p(x) = 0$ as well?

(A) $x = 1 + 2i$

(B) $x = -1 + 2i$

(C) $x = 1 - 2i$

(D) None of the above.

1.11 Determine the following integral: $\int \frac{e^x}{1+e^{2x}} dx$

(A) $\arctan(e^x) + c$

(B) $e^{2x} + c$

(C) $\ln(1 + e^{2x}) + c$

(D) $\frac{e^x}{1+e^{2x}} + c$

1.12 If vectors \mathbf{p} and \mathbf{q} are parallel, which one of the following is true?

(A) $\mathbf{p} \times \mathbf{q} = 1$

(B) $\mathbf{p} \times \mathbf{q} = -1$

(C) $\mathbf{p} \times \mathbf{q} = 0$

(D) $\mathbf{p} \times \mathbf{q} = \frac{1}{2}$

1.13 The asymptotes of $y = 3\arctan(x) - \frac{\pi}{2}$ are

(A) $y = \frac{\pi}{2}$ and $y = -\frac{\pi}{2}$

(B) $y = -\pi$ and $y = 0$

(C) $y = \pi$ and $y = -2\pi$

(D) $y = 0$ and $y = \pi$

1.14 How many terms will the expansion of $x^4(-1 + x)^9$ have?

(A) 8

(B) 10

(C) 9

(D) 11

1.15 The asymptotes of $f(x) = \frac{2x^2 - x - 1}{x^2 - 1}$ are

(A) $y = \frac{1}{2}$ and $x = \pm 1$

(B) $y = 2$ and $x = \pm 1$

(C) $y = \frac{1}{2}$ and $x = -1$

(D) $y = 2$ and $x = -1$

Question 2

[16 marks]

2.1 Use Cramer's rule and determine the value of y if

$$3x + 5y = 7 \text{ and } -x = -1 + y \quad (5)$$

2.2 Solve for x :

(a) $|2x - 1| = x^2 \quad (7)$

(b) $-|2x + 3| \leq -7 \quad (4)$

Question 3

[15 marks]

3.1 Given: $f(x) = 4x^4 - 11x^2 - 3$, with $x = \sqrt{3}$ a zero of the function f .

Factorize f completely in the complex number system. (5)

3.2 Given: $a = -2 + 2i$ and $b = -8i$.

(a) Write a and b in polar form. (4)

(b) Use De Moivre's theorem and determine a^2b . Do all calculations in polar form and give the answer in rectangular form. (6)

Question 4**[12 marks]**

An infectious virus spreads in a village with a population of 10 000. After t days, the number of people infected with the virus is given by $P(t) = \frac{10\,000}{5+1245e^{-0,97t}}$

- 4.1 How many people were originally infected with the virus? (2)
- 4.2 After how many days will 1000 people be infected with the virus?
Give your answer as an integer. (4)
- 4.3 What is the maximum number of people who will contract the virus? (1)
- 4.4 Differentiate the function. (It is not necessary to simplify your answer.) (3)
- 4.5 Describe in words what the derivative function, $P'(t)$ describes. (2)

Question 5**[18 marks]**

- 5.1 Determine the third and the fourth terms in the expansion of $\frac{1}{(1-x)^3}$.
Simplify your answer. (5)
- 5.2 Determine the twelfth term in the expansion of $\left(\frac{1}{x^2} + x\right)^{15}$. (4)
- 5.3 Use mathematical induction and show that the following statement is true for all $n \in \mathbb{N}$: $1 - 1 - 3 + \dots + (3 - 2n) = 2n - n^2$ (9)

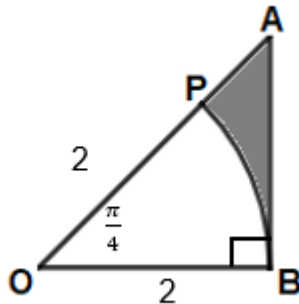
Question 6**[12 marks]**

- 6.1 Given: Vector $\mathbf{u} = (1; 0; 2)$
- (a) Determine the direction cosines of \mathbf{u} . (2)
- (b) Hence, calculate the angle that vector \mathbf{u} make with the z -axis. (2)
- 6.2 The points $P = (2; -1; -1)$, $S = (-1; 0; 2)$ and $T = (0; 5; 1)$ are given.
- (a) Write down the vectors PS and ST . (2)
- (b) Determine whether PS and ST are perpendicular to each other.
Show all your calculations. (4)
- (c) The vector $M = -16i - 16k$ is perpendicular to PS and ST . Determine the size of the area of the parallelogram formed between PS and ST .
Give your answer in surd form. (2)

Question 7**[15 marks]**

7.1 The sketch below shows sector OPB with centre O and a radius of 2cm.

$$\widehat{AOB} = \frac{\pi}{4} \text{ radians and } \widehat{ABO} = \frac{\pi}{2}.$$



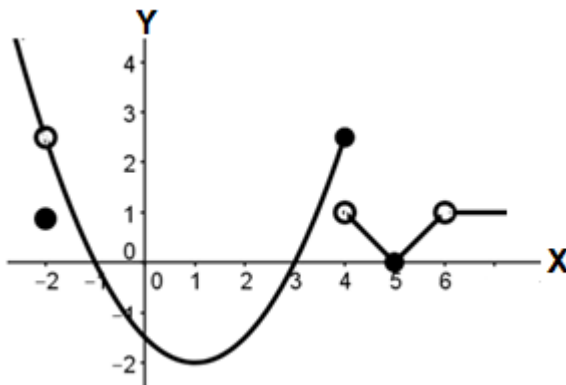
- (a) Motivate why $AB = 2\text{cm}$. (2)
- (b) Determine the arc length PB . (2)
- (c) Determine the perimeter of the shaded area ABP .
Give your (final) answer as a decimal number, rounded to one decimal place. (4)

7.2 Use the answer sheet and sketch the graph of $y = \arccos\left(\frac{x}{2}\right) - \frac{2\pi}{3}$.

Clearly indicate the intercepts with the axes, as well as the starting and ending points of the graph. (7)

Question 8**[20 marks]**

8.1 The graph of $y = f(x)$ is given below:



For which value(s) of x will

- (a) the function f has a removable discontinuity. Motivate your answer. (6)
- (b) the function f has a jump discontinuity. Motivate your answer. (6)

8.2 In each of the following cases, state whether the statement is true or false:

- (a) A continuous function will also be differentiable.
- (b) If a function $y = f(x)$ is differentiable at a point $x = a$,
then $\lim_{x \rightarrow a^-} f'(x) = \lim_{x \rightarrow a^+} f'(x)$
- (c) The x -coordinate of the stationary point of a function f , is also the derivative function's x -intercept.
- (d) If $f'(x) < 0$ in the interval $(a; b)$, the function f will increase in the interval $(a; b)$.
- (e) The inflection point of the function f , can be used to determine the x -coordinate of the stationary point of the first derivative.
- (f) If the function f is concave down in the interval $(a; b)$, then $f''(x) > 0$ in the interval $(a; b)$.
- (g) A function will never intersect a horizontal or oblique asymptote. (14)

Question 9

[14 marks]

9.1 Use the Newton- Rhapson method and determine the x -intercept of $f(x) = \sqrt[5]{x^3} - x + 1$.

- Use $x_0 = 3$ as a first approximation.
- Clearly show the formula you are working with.
- Write down the second approximation (x_1) to six (6) decimal places.
- Write down the final answer to five (5) decimal places. (5)

9.2 Use implicit differentiation and determine the derivative of $2^{7y} = \cos^2(7x) + \pi y$ (9)

Question 10

[17 marks]

Given: $f(x) = \frac{x^3+4}{x^2} = x + \frac{4}{x^2}$

- 10.1 Determine the asymptotes of the function f and name the types of asymptotes. (4)
- 10.2 Determine the coordinates of the stationary point of the function f . (5)
- 10.3 Explain how the second derivative can be used to determine that the stationary point, calculated in 10.2, is a minimum. (It is not necessary to do the calculation.) (2)
- 10.4 Determine the x -intercept of the function f . (1)
- 10.5 It is given that the function f increases for $x < 0$. Use the answer sheet and draw the graph of f . Clearly show all the above values on your sketch. (5)

Question 11**[11 marks]**

11.1 Determine the coordinates of the inflection point of $f(x) = \sqrt[5]{x^{11}} + e$.

Give your answer in terms of e .

(6)

11.2 Motivate why your answer in 11.1 is an inflection point.

(5)

Question 12**[20 marks]**

12.1 Determine the derivative of $f(x) = \left[\frac{1}{\pi x} - \operatorname{cosec} x \right] \left[\ln(5x) + b \operatorname{gcos} \left(-\frac{x}{2} \right) \right]$

(6)

12.2 Determine the following integrals:

(a) $\int (2^{4x} + 2x^4) dx$

(4)

(b) $\int \left[\frac{2}{7x-1} - (7x-1)^2 \right] dx$

(5)

12.3 Determine the volume of the solid of revolution formed when

the surface below $y = \sqrt{x^2 - p}$ rotates about the x -axis, between $x = 2$ and $x = 4$.

Give your answer in terms of p .

(5)

Total: 200