VRAAG 1

1.1	A	**B**	C	D
1.2	A	B	**C**	D
1.3	A	B	C	**D**
1.4	A	B	**C**	D
1.5	A	B	**C**	D
1.6	**A**	B	C	D
1.7	A	**B**	C	D
1.8	A	B	C	**D**
1.9	A	B	C	**D**
1.10	A	B	**C**	D
1.11	A	B	C	**D**
1.12	A	**B**	C	D
1.13	**A**	B	C	D
1.14	A	B	C	**D**
1.15	**A**	B	C	D

Verduidelikings:

1.2 $\left(\frac{1}{3}\right)^2+\left(\frac{2}{3}\right)^2+\left(\frac{2}{3}\right)^2=\frac{9}{9}=1$ **(C)**

1.6 $\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2}(2x)^2=-\frac{4}{9}x^2$ **(A)**

1.11 $x \leq 2$ en $x>0$ **(D)**

1.12 $f'(x)=5x-e^x$, $f''(x)=5-e^x=0$, gee $x=\ln 5$ **(B)**

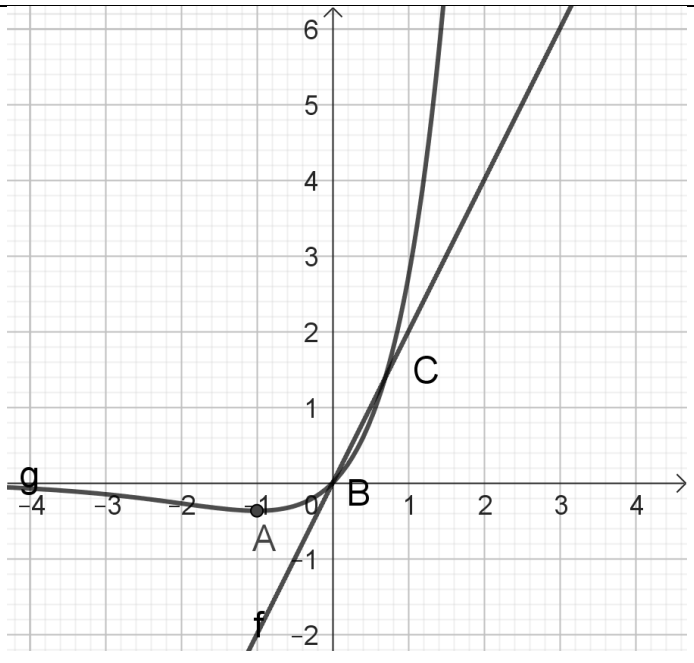
1.13 $\frac{dy}{y}=\sec^2 x\,dx$, dus $\ln y=\tan x$, $y=e^{\tan x}$ **(A)**

1.14 $\frac{d}{dx}\frac{sin^2 x}{2}=\sin x.\cos x$ en $\frac{d}{dx}\left(-\frac{\cos(2x)}{4}\right)=\frac{2\sin 2x}{4}=\frac{4\sin x\cos x}{4}$ **(D)**

1.15 $y'=3x^2+2ax+b$, $y''=6x+2a=0 \quad \therefore a=-3.$

Dus $-6=1-3+b-4 \quad \therefore b=0$ **(A)**

VRAAG 2 **[17 punte]**

2.1(a)	$\mathbf{m(100) = 100e^{-\frac{(\ln 2)\times 100}{1590}}}$ $\mathbf{= 95,73 \approx 96}$ mg	✓ Vervang t=100 ✓Antwoord [2]
2.1(b)	$\mathbf{30 = 100e^{-\frac{(\ln 2)t}{1590}}}$ $\mathbf{\frac{3}{10} = e^{-\frac{(\ln 2)t}{1590}}}$ $\mathbf{\ln 0,3 = -\frac{(\ln 2)t}{1590}}$ $\mathbf{t = \ln 0,3 \times \frac{1590}{\ln 2}}$ $\mathbf{= 2761,78}$ jaar	✓ Vervang ✓ 0,3 ✓Neem $\mathbf{\ln}$ ✓$\mathbf{t}$ alleen ✓Antwoord [5]
2.2 (a)	$\mathbf{(f \circ g)(x) = (e^x - 1 + 1)\ln(e^x - 1 + 1)}$ $\mathbf{= e^x.\ln e^x = e^x.x}$	✓ Vervang ✓ Vereenvoudig [2]
2.2 (b)	$\mathbf{e^x x = 2x}$ $\mathbf{x(e^x - 2) = 0}$ $\mathbf{x = 0}$ *of* $\mathbf{e^x = 2}$ $\mathbf{x = 0}$ *of* $\mathbf{x = \ln 2 = 0,69}$	✓ Stel gelyk ✓Faktoriseer ✓ $\mathbf{x = 0}$ ✓$\mathbf{\ln 2 = 0,69}$ [4]
2.2 (c)		✓ A ✓ $\boldsymbol{B(0;0)}$ ✓ $\boldsymbol{C(\ln 2\,;2\ln 2)}$ ✓ Vorm [4]

VRAAG 3 **[16 punte]**

3.1	$4\left(-1+\sqrt{3}i\right)^n = 4(2\ \text{cis}\frac{2\pi}{3})^n = 4.2^n\ \text{cis}\ (\frac{2n\pi}{3})$ $2^n\left(-\sqrt{3}-i\right)^2 = 2^n\left(2\ \text{cis}\frac{7\pi}{6}\right)^2 = 2^n 2^2\ \text{cis}\ \left(\frac{7\pi}{3}\right)$ *OF* $2^n\left(2\ \text{cis}\frac{-5\pi}{6}\right)^2 = 2^n 2^2\ \text{cis}\ \left(\frac{-5\pi}{3}\right)$ $z = \frac{4.\left(-1-\sqrt{3}i\right)^n}{2^n\left(-\sqrt{3}-i\right)^2} = \frac{4.2^n\ \text{cis}\ (\frac{2n\pi}{3})}{4.2^n\ \text{cis}\left(\frac{7\pi}{3}\right)} = \text{cis}\ \left(\frac{2n\pi}{3}-\frac{7\pi}{3}\right)$ OF $\text{cis}\ \left(\frac{2n\pi}{3}+\frac{5\pi}{3}\right)$	✓✓ Poolvorm t ✓ Tot die mag n ✓ Poolvorm ✓ Tot die mag 2 ✓ Deling Poolvorm [6]
3.2	$\mathbf{z^3 = 4\sqrt{2}cis\left(\frac{3\pi}{4}\right)}$ $\mathbf{z_n = \sqrt[3]{4\sqrt{2}}cis\frac{\frac{3\pi}{4}+2k\pi}{3}, k = 0; 1; 2}$ $\boldsymbol{OF} \quad \boldsymbol{z_n = 2^{\frac{5}{6}}\ \text{cis}\frac{\frac{3\pi}{4}+2k\pi}{3}, k = 0; 1; 2}$ $\mathbf{z_0 = \sqrt[3]{4\sqrt{2}}cis\frac{3\pi}{12}}$ $\quad \boldsymbol{z_0 = 2^{\frac{5}{6}}\ \text{cis}\frac{3\pi}{12}}$ $\mathbf{z_1 = \sqrt[3]{4\sqrt{2}}cis\frac{11\pi}{12}}$ $\quad \boldsymbol{z_1 = 2^{\frac{5}{6}}\ \text{cis}\frac{11\pi}{12}}$ $\mathbf{z_2 = \sqrt[3]{4\sqrt{2}}cis\frac{19\pi}{12}}$ $\quad \boldsymbol{z_2 = 2^{\frac{5}{6}}\ \text{cis}\frac{19\pi}{12}}$	✓ $\mathbf{4\sqrt{2}cis\left(\frac{3\pi}{4}\right)}$ ✓$\boldsymbol{z_n}$ ✓✓✓ 3 Antwoorde [5]
3.3	$(x+1)^2 = (\pm 4i)^2$ *OF Langdeling* $x^2+2x+17$ 'n faktor $2x^3+ax^2+32x-17 = (x^2+2x+17)(2x-1)$ $a = 4-1 = 3$	✓ Metode ✓✓Faktor ✓ $(2x-1)$ ✓ Antwoord [5]

VRAAG 4 **[19 punte]**

4.1 (a)	A $\left(\frac{3}{2};\frac{2\pi}{3}\right)$ B$(-\frac{1}{2};-\frac{\pi}{3})$	✓✓ A ✓✓ B [4]
4.1 (b)	$x \in \left[-\frac{\pi}{3};\frac{2\pi}{3}\right]$	✓ ✓Beperkings [1]
4.1 (c)	$h(x) = bgsin\left(x-\frac{1}{2}\right)+\frac{\pi}{6}+ln(x-1)$ $h'(x) = \frac{1}{\sqrt{1-\left(x-\frac{1}{2}\right)^2}}+\frac{1}{x-1}$ $x_{n+1} = x_n - \frac{bgsin\left(x-\frac{1}{2}\right)+\frac{\pi}{6}+ln(x-1)}{\frac{1}{\sqrt{1-\left(x-\frac{1}{2}\right)^2}}+\frac{1}{x-1}}$ $x_0 = 1,2;\ x_1 = 1,24850;\ x_2 = 1,25264;\ x_3 = 1,25266$ $x \approx 1,2527$	✓Maak een funksie ✓ Differensieer ✓ Vorm ✓Antwoord ✓Regte desimale [5]
4.2	$x < -2$ *OF* $x \geq -2$ $-x-2 = -\frac{x}{4}+4$ $x+2 = -\frac{x}{4}+4$ $3x = -24$ $5x = 8$ $x = -8$ *OF* $x = \frac{8}{5}$	✓ Beperkings ✓✓ Twee vergelykings ✓✓ Twee antwoorde [5]
4.3	$z = \begin{vmatrix} 2 & 1 & 0 \\ 1 & -2 & t \\ 4 & 3 & 1 \end{vmatrix} \div 3 = -5$ $2(-2-3t)-(1-4t) = -15$ $-2t-5 = -15;\ -2t = -10\ ,\ t = 5$	✓✓ Determinant/3 = -5 ✓ Vereenvoudig ✓ Antwoord [4]

VRAAG 5 **[20 punte]**

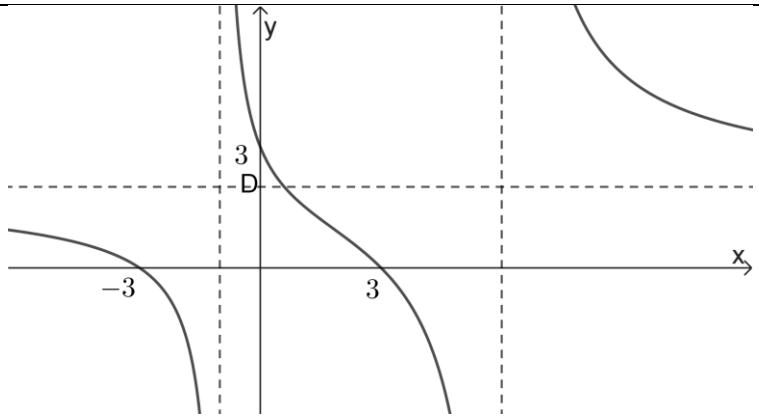
5.1 (a)	$\|\mathbf{AB}\| = \sqrt{(3-2)^2 + (-1+1)^2 + (5+1)^2}$ $\|\mathbf{AB}\| = \sqrt{37}$	✓ Vervang in formule ✓ Antwoord [2]
5.1 (b)	$\theta_y = \mathbf{bgcos}\left(-\frac{1}{\sqrt{35}}\right)$ $\theta_y = 1,74$ rad	✓Formule en -1 ✓$\sqrt{35}$ ✓Antwoord [3]
5.1 (c)	$\mathbf{A} \cdot \mathbf{B} = 6 + 1 - 5 = 2$ $\cos\theta = \frac{\mathbf{A}\cdot\mathbf{B}}{\|A\|\|B\|}$ $= \frac{2}{\sqrt{35}\sqrt{6}}$ $\theta = \mathbf{bgcos}\frac{2}{\sqrt{210}}$ $\theta = 1,43$ rad	✓ $\mathbf{AB}.\mathbf{BC}$ ✓Formule ✓ $\sqrt{6}$ ✓ $\mathbf{bgcos}\frac{2}{\sqrt{210}}$ ✓ Antwoord [5]
5.1(d)	$\mathbf{A} \times \mathbf{B} = \begin{pmatrix} i & j & k \\ 3 & -1 & 5 \\ 2 & -1 & -1 \end{pmatrix}$ $= i(1+5) - j(-3-10) + k(-3+2)$ $= 6i + 13j - k$	✓ $\mathbf{A} \times \mathbf{B}$ ✓✓✓ Kruisproduk [4]
5.2 (a)	$\mathbf{A} = \frac{1}{2}6^2.\theta - \frac{1}{2}4^2.\theta \qquad (\mathbf{A} = \frac{1}{2}\mathbf{r}^2.\theta)$ $= 18\theta - 8\theta = 10\theta$ maar $A = 15\pi$ $\therefore \theta = \frac{15\pi}{10} = \frac{3\pi}{2}$	✓ Formule ✓ Vervang ✓Vereenvoudig ✓ Antwoord [4]
5.2 (b)	$\mathbf{Booglengte} = \mathbf{r\theta}$ Omtrek $= 2 + 2 + 4 \times \frac{3\pi}{2} + 6 \times \frac{3\pi}{2}$ $= 4 + 15\pi$	✓ Vervang in formule ✓Antwoord [2]

VRAAG 6 **[20 punte]**

6.1 (a)	$\lim\limits_{x\to\frac{5\pi^-}{3}} f(x) = \lim\limits_{x\to\frac{5\pi^-}{3}} (-2\ \cos x) = -1$ en $\lim\limits_{x\to\frac{5\pi^+}{3}} f(x) = \lim\limits_{x\to\frac{5\pi^+}{3}} \frac{-\sqrt{3}}{2\pi}\left(x-\frac{2\pi}{3}\right)^2 = -\frac{\sqrt{3}\pi}{2} \approx -2,72$ Nee, nie kontinu nie Sprong	✓ Notasie ✓ Limiet van links ✓ Limiet van regs ✓Antwoord ✓Sprong [5]
6.1 (b)	$\lim\limits_{x\to\frac{5\pi^-}{3}} f'(x) = \lim\limits_{x\to\frac{5\pi^-}{3}} (-2\sin x)$ $= -\sqrt{3}$ en $\lim\limits_{x\to\frac{5\pi^+}{3}} f'(x) = \lim\limits_{x\to\frac{5\pi^+}{3}} \frac{-\sqrt{3}}{2\pi}.2\left(x-\frac{2\pi}{3}\right)^1$ $= -\sqrt{3}$	✓ Afgeleide ✓ Limiet van links ✓ Afgeleide ✓ Limiet van regs ✓ Skryfwyse [5]
6.1 (c)	Nee, want dit is nie kontinu nie. (Alle differensieerbare funksdies is kontinu)	✓ Nee ✓Rede [2]
6.2	Stel $n = 1$: $LK = 1^3 = 1$ en $RK = \frac{1}{4}(1+1)^2 = 1$. Dus bewering is waar vir $n = 1$. Aanvaar bewering waar vir $n = k$: $1^3+2^3+3^3+k^3 = \frac{k^2}{4}(k+1)^2$ (i) Bewys dat die bewering geld vir $n = k+1$: $LK = 1^3+2^3+3^3+k^3+(k+1)^3$ $= \frac{k^2}{4}(k+1)^2+(k+1)^3$ uit (i) $= \frac{(k+1)^2}{4}\left(k^2+4(k+1)\right) = \frac{(k+1)^2}{4}(k^2+4k+4)$ $= \frac{(k+1)^2}{4}(k+2)^2$ $RK = \frac{(k+1)^2}{4}((k+1)+1)^2 = \frac{(k+1)^2}{4}(k+2)^2$ Dus bewering waar vir $n = k+1$ Volgens die beginsel van wiskundige induksie, is die bewering dus waar vir alle $n \in \mathbb{N}$	✓ $n = 1$ ✓ Aanvaar waar ✓Te bewys $n = k+1$, ✓ RK ✓ LK 1e stap ✓ Skei eksponente ✓ Vereenvoudig ✓ Rede [8]

VRAAG 7 **[23 punte]**

7.1 (a)	$\frac{d}{dx}[2^{\text{cosec } 3x}] = 2^{\text{cosec } 3x} \ln 2 \times(-\text{cosec } 3x \cot 3x) \times 3$	✓ $2^{\text{cosec } 3x} \ln 2$ ✓ $-\text{cosec } 3x \cot 3x$ ✓ 3 [3]
7.1 (b)	$g(x) = e^{\tan x.\left(\sqrt[4]{5x^3}\right)}$ $g'(x) = e^{\tan x.\left(\sqrt[4]{5x^3}\right)}(\sec^2 x.\sqrt[4]{5x^3} + \tan x.\frac{1}{4}(5x^3)^{-\frac{3}{4}}.15x^2$ *OF laaste deel* $D_x\left(5^{\frac{1}{4}}.x^{\frac{3}{4}}\right) = \left(5^{\frac{1}{4}}.\frac{3}{4}x^{\frac{-1}{4}}\right)$	✓ $e^{\tan x.\sqrt[4]{5x^3}}$ ✓ $\sec^2 x.\sqrt[4]{5x^3}$ ✓ *Produkreël* $f'g + fg''$ ✓ $\tan x.\frac{1}{4}(5x^3)^{-\frac{3}{4}}$ ✓ $15x^2$ [5]
7.2	$\cot 2y - y\ln x + x = 2$ $-\text{cosec}^2(2y).2.y' - y'\ln x - \frac{y}{x} + 1 = 0$ $-\frac{1}{\frac{1}{2}}.2.y' - y'.0 - \frac{\pi}{8} + 1 = 0 \quad \therefore -4y' = \frac{\pi}{8} - 1$ *OF* $y' = \frac{1-\frac{y}{x}}{2\text{cosec}^2(2y)+\ln x} = \frac{1-\frac{\pi}{8}}{2\text{cosec}^2\left(\frac{\pi}{4}\right)+0} = \frac{1-\frac{\pi}{8}}{4}$ Dus $y' = \frac{1}{4} - \frac{\pi}{32}$ $\therefore a = \frac{1}{4}$ en $b = -\frac{1}{32}$	✓✓$-\text{cosec}^2(2y).2.y'$ ✓ $y'\ln x$ ✓ $-\frac{y}{x} + 1$ ✓ Vervang $(1;\frac{\pi}{8})$ ✓ a en b [6]
7.3	$\Delta x_i = \frac{2}{n}$; $x_i = 2 + \frac{2}{n}.i$ $f(x_i) = 3(2 + \frac{2}{n}.i)^2 - 6(2 + \frac{2}{n}.i)$ $= 12 + 24\frac{i}{n} + 12\frac{i^2}{n^2} - 12 - 12\frac{i}{n} = 12\frac{i^2}{n^2} + 12\frac{i}{n}$ $\sum_{i=1}^{n}\left(12\frac{i^2}{n^2} + 12\frac{i}{n}\right)$ $= -\frac{12}{n^2}\left(\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}\right) + \frac{12}{n}\left(\frac{n^2}{2} + \frac{n}{2}\right)$ $= 4n + 6 + \frac{2}{n} + 6n + 6$ $= 10n + 12 + \frac{2}{n}$ Oppervlakte = basis x hoogte = $(10n + 12 + \frac{2}{n}).\frac{2}{n}$ $= 20 + \frac{24}{n} + \frac{4}{n^2}$ $\int_2^4 (3x^2 - 6x)\,dx = \lim_{n\to\infty}\left(20 + \frac{24}{n} + \frac{4}{n^2}\right) = 20$	✓ Δx_i ✓ x_i ✓ $f(x_i)$ ✓ sigma i^2 ✓sigma i ✓Vereenvoudig ✓ b x h ✓lim ✓Antwoord (9)

8.1	$f(x) = \frac{2x^2-18}{x^2-5x-6}$. $2x^2 - 18 = 0$ $x = \pm 3$ $y = 3$	✓ Stel teller = 0 ✓Antwoorde **x** ✓Antwoord **y** [3]
8.2	$x^2 - 5x - 6 = 0$ $x = -1$; $x = 6$ $y = 2$	✓Stel noemer = 0 ✓ Vertikale asimptote ✓ Horisontale asimptoot [3]
8.3	$f'(x) = \frac{(4x)(x^2-5x-6)-(2x^2-18)(2x-5)}{(x^2-5x-6)^2} = 0$ $4x^3 - 20x^2 - 24x - 4x^3 + 10x^2 + 36x - 90 = 0$ $-10x^2 + 12x - 90 = 0$ $\Delta = 144 - 4(-10)(-90) < 0$ *(OF formule)* *Dus geen reële oplossing en geen draaipunte.*	✓✓Differensieer = 0 ✓ Vereenvoudig ✓Delta of formule [4]
8.4	$\frac{2x^2-18}{x^2-5x-6} = 2$ $2x^2 - 18 = 2x^2 - 10x - 12$ $x = \frac{3}{5}$	✓ Vereenvoudig ✓ Antwoord [2]
8.5		✓ Afsnitte ✓Asimptote ✓Punt D ✓✓ Vorm [5]

9.1	$\int \left(\left(\frac{8}{(4x-2)\ln 5}\right)+10\right)dx$ $=\frac{8\log_5(4x-2)}{4}+10x+k$ $OF\ =\frac{8}{\ln 5}\times\frac{\ln(4x-2)}{4}+10x+k$	✓✓ $\log_5(x-2)$ ✓$\frac{8}{4}$ of 2 [3]
9.2	Stel $\frac{3x^2+3x+7}{(x^2+1)(x-2)}=\frac{Ax+B}{x^2+1}+\frac{C}{x-2}$ $3x^2+3x+7=(Ax+B)(x-2)+C(x^2+1)$ Stel $x=2$: $25=C(5);\ C=5$ x^2: $3=A+5 \quad A=-2$ k: $7=-2B+5 \quad B=-1$ $\int\left(-\frac{2x+1}{x^2+1}+\frac{5}{x-2}\right)dx$ $=\int\left(-\frac{2x}{x^2+1}-\frac{1}{x^2+1}+\frac{5}{x-2}\right)dx$ $=-\ln(x^2+1)-\text{bgtan}\,x+5\ln\lvert x-2\rvert+k$	✓ Pars breuke ✓ Maal kgv ✓✓ A, B en C ✓ Skei breuk ✓✓✓Antwoord [8]
9.3	Stel $f=x^2$ *en* $g'=e^x$ Dan $f'=2x$ en $g=e^x$ $\int x^2.e^x\,dx=x^2e^x-\int 2xe^x\,dx$ Stel $f=2x$ en $g'=e^x$ Dan $f'=2$ en $g=e^x$ $\int x^2.e^x\,dx=x^2e^x-2xe^x+\int 2e^x\,dx$ $\int x^2.e^x\,dx=x^2e^x-2xe^x+2e^x+k$	✓ **f** en **g**′ ✓ **f**′en **g** ✓ integraal ✓ **f en g**′ ✓ **f**′**en g** ✓ integraal [6]
9.4	$\frac{1}{2}\int\left(\sin 2x.\frac{1}{\sqrt{x}}+4\sqrt{x}.\cos 2x\right)dx$ $=\int\left(\sin 2x.\frac{1}{2\sqrt{x}}+\sqrt{x}.2\cos 2x\right)dx$ Stel $f=\sin 2x$ en $g=\sqrt{x}$ Dan $f'=2\cos 2x$ *en* $g'=\frac{1}{2\sqrt{x}}$ Dus $\int\left(\sin 2x.\frac{1}{2\sqrt{x}}+\sqrt{x}.\cos 2x\right)dx=\int(fg'+f'g)dx$ $=\int\frac{d}{dx}(fg)dx$ $=\sin 2x.\sqrt{x}$	✓ **f** en **g** ✓ **f**′ en **g**′ ✓ Vervang in integraal ✓ Antwoord (*Antwoord alleen volpunte*) [4]

10.1 (a)	$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\left(4\sin^2 x - \sec^2\left(x-\frac{\pi}{6}\right)\right)dx$ $= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\left(4\left(\frac{1-\cos 2x)}{2}\right) - \sec^2\left(x-\frac{\pi}{6}\right)\right)dx$ $= (2x - \sin 2x - \tan\left(x-\frac{\pi}{6}\right)\Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}}$ $= \left(\pi - \sin\pi - \tan\left(\frac{\pi}{3}\right)\right) - \left(\frac{\pi}{3} - \sin\frac{\pi}{3} - \tan 0\right)$ $= \pi - 0 - \sqrt{3} - \frac{\pi}{3} + \frac{\sqrt{3}}{2} + 0 = \frac{2\pi}{3} - \frac{\sqrt{3}}{2}$ $\therefore a = \frac{2}{3}, \quad b = -\frac{1}{2} \quad en \quad c = 3$	✓Integraal reg ✓ Trig. identiteit ✓✓ **Integrasie** ✓ Vervang ✓Vereenvoudig ✓Antwoord [7]
10.1 (b)	$Vol = \pi\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}((2\sin x)^4 - \sec^4(x-\frac{\pi}{6}))dx$	✓ $\pi\int f^2 - g^2$ ✓ f^2 ✓ g^2 (3)
10.2	$\int_0^a \frac{1}{\sqrt{3+4x-4x^2}}dx = \frac{\pi}{6}$ $3 + 4x - 4x^2 = 4 - (4x^2 - 4x + 1) = 4(1 - \left(\frac{2x-1}{2}\right)^2)$ $\int_0^a \frac{1}{\sqrt{3+4x-4x^2}}dx = \frac{1}{2}\int_0^a \frac{1}{\sqrt{1-\left(x-\frac{1}{2}\right)^2}}dx$ $= \frac{1}{2}\text{bgsin}(x-\frac{1}{2})\Big|_0^a = \frac{1}{2}\text{bgsin}(a-\frac{1}{2}) - \frac{1}{2}\text{bgsin}(-\frac{1}{2})$ $\therefore \text{bgsin}(a-\frac{1}{2}) + \frac{\pi}{6} = 2\times\frac{\pi}{6} \quad \therefore a - \frac{1}{2} = \sin\frac{\pi}{6} = \frac{1}{2}$ $\therefore a = 1$	✓✓ Voltooi vierkant ✓ Integreer ✓✓ Vervang grense ✓ Vereenvoudig ✓ Antwoord [7]