α MATHEMATICS

Alpha Mathematics
PRELIMINARY EXAM

September 2025 **Time: 3 hours**

Grade 12 **Total: 200 marks**

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the question paper:

1. This question paper consists of seven pages and a two-page answer sheet . The formula sheet contains three pages.
2. Answer ALL 11 questions on this question paper.
3. Question 1 consists of 15 multiple choice questions . Answer them on the answer sheet .
4. Write your name on the answer sheet of the question paper.
5. Non- programmable calculators may be used, unless otherwise stated in a specific question.
6. Unless otherwise specified, all answers, where applicable, must be rounded correctly to **two decimal places .**
7. The diagrams in the question paper are not necessarily drawn to scale.
8. **All angles are given in radians.** Answers should be given in radians where applicable.
9. Clearly indicate all necessary calculations for each question. The correct answer in itself will not necessarily lead to full marks.
10. Write neatly and legibly.

QUESTION 1 [30 MARKS]

- Answer this question **on the answer sheet ,** by placing an X (cross) on A, B, C or D.
- Please **DO NOT** detach this page from the question paper.
- These questions count 2 marks each.

1.1 The product of $2\text{cis}\left(\frac{\pi}{6}\right)$ and $6\text{cis}\left(\frac{\pi}{2}\right)$ is

(A) $3\text{cis}\left(\frac{\pi}{3}\right)$ **(B)** $12\text{cis}\left(\frac{2\pi}{3}\right)$

(C) $8\text{cis}\left(\frac{2\pi}{3}\right)$ **(D)** $12\text{cis}\left(\frac{\pi}{3}\right)$

1.2 If $|2x+1| = -x-1$, which one of the following is correct?

(A) $x = -\frac{2}{3}$ or $x = 0$ **(B)** $x = 0$

(C) $x = -\frac{2}{3}$ **(D)** No solution

1.3 The first three terms in the binomial expansion of $(1-2x)^5$ are

(A) $1-10x+40x^2$ **(B)** $1-10x-80x^2$

(C) $1-10x-40x^2$ **(D)** $1-10x-40x^3$

1.4 For what values of x will the expansion of $\frac{1}{2+x}$ converge?

(A) $|x| > 2$ **(B)** $|x| > \frac{1}{2}$

(C) $|x| < 2$ **(D)** $|x| < \frac{1}{2}$

1.5 $f(x) = \begin{cases} x^2-1 & \text{if } x < 2 \\ 3 & \text{if } x > 2 \end{cases}$

What type of discontinuity will the function f have at $x = 2$?

(A) removable discontinuity **(B)** asymptotic discontinuity

(C) jump discontinuity **(D)** none of the above.

1.6 The function $g(x) = \frac{3x^2+1}{x}$ has the following asymptote:

(A) Horizontal: $y = 3$ **(B)** Horizontal: $y = 1$

(C) Oblique: $y = x$ **(D)** Oblique: $y = 3x$

1.7 If a function has a maximum at $x = 1$, then

(A) $f''(1) > 0$ **(B)** $f''(1) < 0$

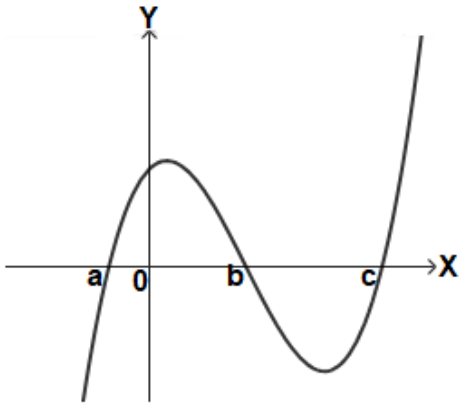
(C) $f''(1) = 0$ **(D)** None of the above

1.8 The range of $y = \arcsin\left(\frac{x}{3}\right) + \frac{\pi}{2}$ is

(A) $y \in (0; \pi)$ **(B)** $y \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$

(C) $y \in [0; \pi]$ **(D)** $y \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$

1.9 The graph below is a representation of $y = f''(x)$. The x-intercepts are at $x = a$, $x = b$ and $x = c$.



Where will the function f bend concave up?

(A) $a < x < b$ **(B)** $x < a$ and $b < x < c$

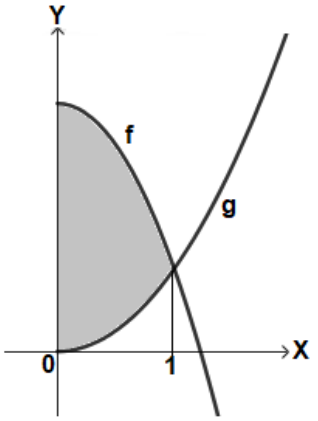
(C) $b < x < c$ **(D)** $a < x < b$ and $x > c$

1.10 The equation of the tangent to $y = \frac{1}{\sqrt{x}}$ at $x = 4$ is

(A) $y = \frac{x}{2}$ **(B)** $y = \frac{x}{16} + \frac{1}{4}$

(C) $y = -\frac{x}{2}$ **(D)** $y = \frac{-x}{16} + \frac{3}{4}$

1.11



The area between the graphs of f and g rotates about the x-axis between $x = 0$ and $x = 1$. The volume of this resulting body can be calculated by

(A) $\pi \int_0^1 \left[\left(f(x)\right)^2 - \left(g(x)\right)^2\right] dx$ **(B)** $\pi \int_0^1 [f(x) - g(x)]dx$

(C) $\pi \int_0^1 [(f(x) - g(x))^2]dx$ **(D)** $\pi \int_0^1 [f(x) + g(x)]dx$

1.12 Determine $\frac{dy}{dx}$ if $y = \log_3(x^4 - 1)$

(A) $\frac{dy}{dx} = \frac{1}{x^4-1}$ **(B)** $\frac{dy}{dx} = \frac{4x^3}{(x^4-1)\ln 3}$

(C) $\frac{dy}{dx} = \frac{x^4-1}{\ln 3}$ **(D)** $\frac{dy}{dx} = \frac{x^4-1}{4x^3}$

1.13 Write as one logarithm: $\ln x - 2\ln y - \ln z$

(A) $\ln\left(\frac{xz}{2y}\right)$ **(B)** $\ln\left(\frac{x}{y^2z}\right)$

(C) $\ln\left(\frac{xz}{y^2}\right)$ **(D)** $\ln\left(\frac{y^2z}{x}\right)$

1.14 Simplify: $i^{13} + i^{23} - i^6$

(A) -1 **(B)** $-i$

(C) 1 **(D)** i

1.15 What is the vertical component of a vector with a magnitude of 10, in the direction of $\frac{\pi}{3}$ radians, north of east?

(A) $5\sqrt{3}$ **(B)** 5

(C) $10\sqrt{3}$ **(D)** 10

QUESTION 2 [21 MARKS]

2.1 Given: $f(x) = x^3 + x^2 - 7x - 3$

Factorize the function f completely in the real number system, if $x = 1 + \sqrt{2}$ is a zero of f. (4)

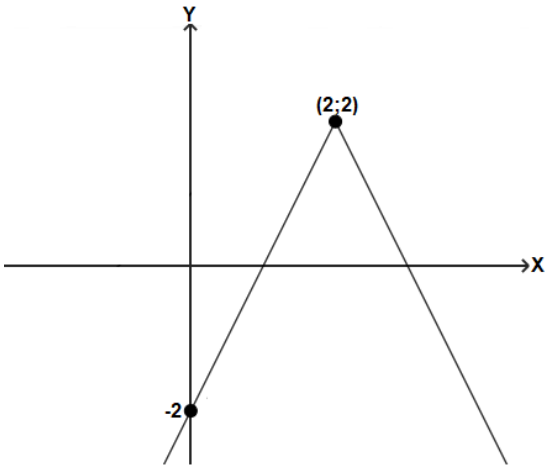
2.2 The complex numbers $p = 6e^{\pi i}$ and $t = -\sqrt{3} - i$ are given.

(a) Convert p and t into polar form. (3)

(b) Use De Moivre's theorem and calculate $\left(\frac{p}{t}\right)^3$. Give the answer in rectangular form. (6)

(c) Calculate $\sqrt{t}$ (3)

2.3 The sketch of $f(x) = a|bx + c| + b$ is given below:



The function f has an y-intercept at $y = -2$ and a salient point at $(2; 2)$.

Calculate the values of a, b and c. (5)

QUESTION 3 [22 MARKS]

3.1 In a specific electrical circuit, the voltage, V (volts) across a capacitor, is given by $V(t) = 5{,}9(1 - e^{-2kt})$. In this formula, k is a positive constant and the variable t refers to the number of seconds that have elapsed since the circuit was turned on.

(a) If $t = 3$, the voltage across the capacitor will be equal to 2,7 volts. Calculate the value of k, correct to three decimal places. (5)

(b) Use $k = 0{,}1$ and determine the rate at which the voltage increases when $t = 2$. Give the answer correct to two decimal places. (5)

3 .2 Given: $f(x) = 2\ln(x+1) + 3$

(a) Calculate the y-intercept of the function f. (2)

(b) Use the Newton-Rhapson method and determine the x-intercept of the function f.

- Use $x_0 = -0{,}8$ as the first approximation.
- Clearly show the formula you are working with.
- Write down the second approximation (x_1) to five (5) decimal places.
- Write down the final answer to three (3) decimal places. (5)

(c) Determine the asymptote of f. (1)

(d) Draw a graph of $|f(x)|$. Clearly indicate all the above points. (4)

QUESTION 4 [15 MARKS]

4.1 Use mathematical induction and show that the following statement is true for all $n \in \mathbb{N}$: (10)

$$\sum_{r=1}^{n} 2^{2-r} = 4 - 2^{2-n}$$

4.2 The fourth term, in the binomial expansion of $\sqrt{1+ax}$, is $-\frac{x^3}{128}$. Determine the value of a. (5)

QUESTION 5 [16 MARKS]

5.1 Given: $f(x) = 2\arctan(x-1) - \frac{\pi}{3}$

(a) Determine the inverse of the function f. (3)

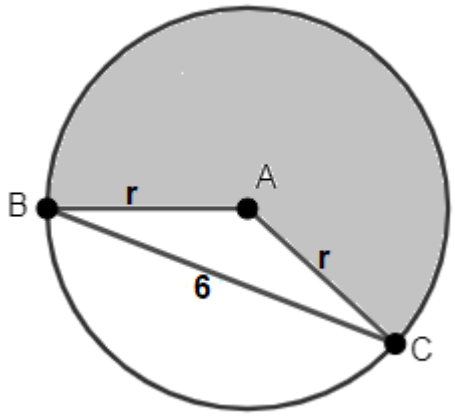
(b) Determine the range of f. (2)

(c) The graph of f undergoes the following changes:

- Reflect around the y-axis.
- Move $\frac{\pi}{6}$ units down
- Reduce by factor 3.

Give the new function, $g(x)$, that arises in this way. (3)

5.2 A circle with centre A and radius r is given below. The circumference of the circle is 20cm. Chord BC is 6cm.



(a) Show that $r = \frac{10}{\pi}$. (2)

(b) Determine the size of the obtuse angle BAC. Give your answer correct to three decimal places. (3)

(c) Determine the area of the shaded portion. Assume that the obtuse angle BAC=2,5 radians. (3)

QUESTION 6 [11 MARKS]

6.1 Points P(1;-1;3) and T(-1;2;0) are given. Determine the distance between P and T. Give your answer in surd form. (2)

6.2 (a) Determine a vector, $\boldsymbol{w}$, that is perpendicular to the vectors $\boldsymbol{u} = i + 2j - k$ and $\boldsymbol{v} = -i + j$. (5)

(b) Are vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ parallel? Motivate your answer. (2)

(c) Determine the size of the angle between $\boldsymbol{u}$ and $\boldsymbol{v}$, if $|\boldsymbol{u}| = \sqrt{6}$ and $|\boldsymbol{v}| = \sqrt{2}$ (2)

QUESTION 7 [22 MARKS]

7.1 Given: $f(x) = \begin{cases} 2^{3x} + 1 & \text{if } x \leq p \\ 2 & \text{if } x > p \end{cases}$

(a) The function f is continuous at $x = p$. Show that $p = 0$. (4)

(b) Is the function differentiable at $x = p$? Determine the derivative of f and fully motivate your answer. (6)

7.2 Determine the derivatives of the following:

(a) $f(x) = \tan\big(\arccos(2x)\big)$ (3)

(b) $2xy^3 + \frac{1}{5y} = ex$ (Hint: Use implicit differentiation.) (9)

QUESTION 8 [15 MARKS]

Given: $f(x) = \frac{2x^2+x-1}{x^2-x-2}$

8.1 Determine and classify the asymptotes of the function f. (4)

8.2 Show algebraically that the function decreases for all $x \in \mathbb{R}$. (5)

8.3 It is given that $f\left(\frac{1}{2}\right) = f^{-1}\left(\frac{1}{2}\right) = 0$. The function has no stationary points. Use the answer sheet and sketch the graph of f. (6)

QUESTION 9 [16 MARKS]

Given: $f(x) = (2x-3)^5 + e$

9.1 Determine the coordinates of the stationary point of the function f. Give the answer in terms of e. (7)

9.2 Use the second derivative to classify the stationary point and fully motivate your answer. (9)

QUESTION 10 [16 MARKS]

10.1 Determine the following integrals:

(a) $\int(\sin^2(3x) - \text{cosec}^2 x)dx$ (5)

(b) $\int x^2 . 5^{4x^3-1} dx$ (3)

(c) $\int(e^{3x-1} + \ln 2)dx$ (3)

10.2 (a) The function $f(x) = \frac{-x^2+4x+1}{(2x+1)(1+x^2)}$ can be written as $f(x) = \frac{A}{2x+1} + \frac{2}{1+x^2}$. Show that $A = -1$. (2)

(b) Hence, determine $\int f(x)\,dx$ (3)

QUESTION 11 [16 MARKS]

11.1 Use **integration by parts** and show that the area between the graph of $y = (2x+1)\ln x$, the x-axis and the points $x = 1$ and $x = e$ is equal to $\frac{1}{2}(e^2+3)$. **Show all calculations.** (7)

11.2 Determine $\int_1^3(-x^2+1)dx$ with a Riemann sum . (9)

Total: 200